

Find the average rate of change of the function over the given interval. Compare this average rate of change with the instantaneous rates of change at the endpoints of the interval.

1a) $f(t) = t^2 - 7$, $[3, 3.1]$ **1b)** $f(t) = 4t + 5$, $[1, 2]$ **1c)** $f(x) = \frac{-1}{x}$, $[1, 2]$

$$\frac{f(3.1) - f(3)}{3.1 - 3}$$

$$\text{Avg. ROC} = 6.1$$

$$f'(x) = 2x$$

$$f'(3.1) = 6.2$$

$$f'(3) = 6$$

2a) Use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects to answer the following questions.

A silver dollar is dropped from the top of a building that is 1362 feet tall.

- Determine the position and velocity functions for the coin.
- Determine the average velocity on the interval $[1, 2]$.
- Find the instantaneous velocities when $t = 1$ and $t = 2$.
- Find the time required for the coin to reach ground level.
- Find the velocity of the coin at impact.

$$(a) \quad s(t) = -16t^2 + 1362$$

$$v(t) = -32t$$

$$(d) \quad -16t^2 + 1362 = 0$$

$$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

$$(b) \quad \frac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48 \text{ ft/sec}$$

$$(c) \quad v(t) = s'(t) = -32t$$

$$\text{When } t = 1: v(1) = -32 \text{ ft/sec}$$

$$\text{When } t = 2: v(2) = -64 \text{ ft/sec}$$

$$(e) \quad v\left(\frac{\sqrt{1362}}{4}\right) = -32\left(\frac{\sqrt{1362}}{4}\right) = -8\sqrt{1362} \approx -295.242 \text{ ft/sec}$$

2b) A ball is thrown straight down from the top of a 220-foot building with an initial velocity of (-22) feet per second.

What is its velocity after 3 seconds?

What is its velocity after falling 108 feet?

3a)

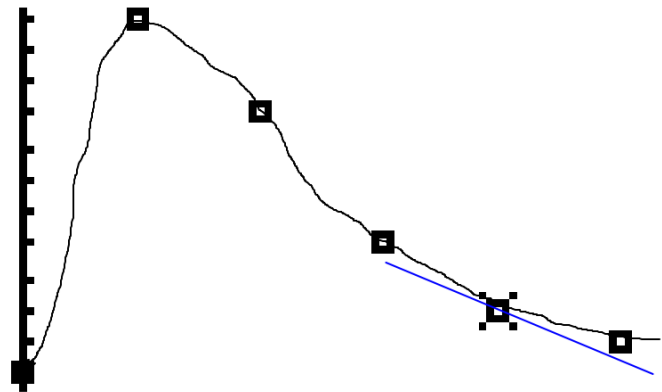
t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.

- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(a)

$W'(12)$ means slope of the function when $t = 12$ days. We can estimate that using average rate of change (slope between) over $[9, 12]$ which is $-2/3$ °C/day



Note: average roc over $[12, 15]$ ($-1/3$ °C/day)

Would also be an appropriate answer!

(b) Using a calculator, $\frac{d}{dt}(20+10te^{-t/3})|_{t=12}$
 -0.54946917

So, on the 12th day the water is cooling -0.549 °C/day

3b)

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15

The height of a tree at time t is given by a twice-differentiable function H , where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

- (a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.
- (b) If the function $H(t) = -0.057t^3 + 1.014t^2 - 3.378t + 4.623$ is a good representation for this data, use your graphing calculator to check your estimate from part (a).

4) AP MULTIPLE CHOICE EXAMPLES

- 1) A particle moves along the x -axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest?
- (A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3
- 2) The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?
- (A) 0 (B) 2 (C) 4 (D) 8 (E) 12

3) If the position of a particle on the x -axis at time t is $-5t^2$, then the average velocity of the particle for $0 \leq t \leq 3$ is

- (A) -45 (B) -30 (C) -15 (D) -10 (E) -5

4) GRAPHING CALCULATOR NEEDED

A particle moves along a line so that at time t , where $0 \leq t \leq \pi$, its position is given by

$s(t) = -4 \cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

- (A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13