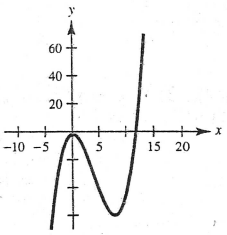
47. $f(x) = \frac{1}{4}x^3 - 3x^2 + \frac{3}{4}x - 2$

$f'(x) = \frac{3}{4}x^2 - 6x + \frac{3}{4}$

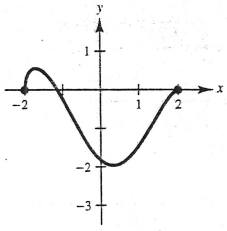


Let $x_1 = 12$.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	12.0000	7.0000	36.7500	0.1905	11.8095
2	11.8095	0.2151	34.4912	0.0062	11.8033
3	11.8033	0.0015	34.4186	0.0000	11.8033

Approximation: $x \approx 11.803$

48. $f(x) = \sqrt{4 - x^2}\sin(x - 2)$



Domain: $[-2, 2]$

$x = -2$ and $x = 2$ are both zeros.

$$f'(x) = \sqrt{4 - x^2}\cos(x - 2) - \frac{x}{\sqrt{4 - x^2}}\sin(x - 2)$$

Let $x_1 = -1$.

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1	−1.0000	−0.2444	−1.7962	0.1361	−1.1361
2	−1.1361	−0.0090	−1.6498	0.0055	−1.1416
3	−1.1416	0.0000	−1.6422	0.0000	−1.1416

Zeros: $x = \pm 2, x \approx -1.142$

Section 3.9 Differentials

1. $f(x) = x^2$

$f'(x) = 2x$

Tangent line at $(2, 4)$: $y - f(2) = f'(2)(x - 2)$

$$y - 4 = 4(x - 2)$$

$$y = 4x - 4$$

x	1.9	1.99	2	2.01	2.1
$f(x) = x^2$	3.6100	3.9601	4	4.0401	4.4100
$T(x) = 4x - 4$	3.6000	3.9600	4	4.0400	4.4000

2. $f(x) = \frac{1}{x^2}$

$f'(x) = -\frac{2}{x^3}$

Tangent line at $\left(2, \frac{1}{4}\right)$:

$$y - f(2) = f'(2)(x - 2)$$

$$y - \frac{1}{4} = -\frac{1}{4}(x - 2)$$

$$y = -\frac{1}{4}x + \frac{3}{4}$$

x	1.9	1.99	2	2.01	2.1
$f(x) = \frac{1}{x^2}$	0.2770	0.2525	0.2500	0.2475	0.2268
$T(x) = -\frac{1}{4}x + \frac{3}{4}$	0.2750	0.2525	0.2500	0.2475	0.2250

3. $f(x) = x^5$

$f'(x) = 5x^4$

Tangent line at $(2, 32)$: $y - f(2) = f'(2)(x - 2)$

$y - 32 = 80(x - 2)$

$y = 80x - 128$

x	1.9	1.99	2	2.01	2.1
$f(x) = x^5$	24.7610	31.2080	32	32.8080	40.8410
$T(x) = 80x - 128$	24.0000	31.2000	32	32.8000	40.0000

4. $f(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

Tangent line at $(2, \sqrt{2})$:

$y - f(2) = f'(2)(x - 2)$

$y - \sqrt{2} = \dfrac{1}{2\sqrt{2}}(x - 2)$

$y = \dfrac{x}{2\sqrt{2}} + \dfrac{1}{\sqrt{2}}$

x	1.9	1.99	2	2.01	2.1
$f(x) = \sqrt{x}$	1.3784	1.4107	1.4142	1.4177	1.4491
$T(x) = \dfrac{x}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}$	1.3789	1.4107	1.4142	1.4177	1.4496

5. $f(x) = \sin x$

$f'(x) = \cos x$

Tangent line at $(2, \sin 2)$:

$y - f(2) = f'(2)(x - 2)$

$y - \sin 2 = (\cos 2)(x - 2)$

$y = (\cos 2)(x - 2) + \sin 2$

x	1.9	1.99	2	2.01	2.1
$f(x) = \sin x$	0.9463	0.9134	0.9093	0.9051	0.8632
$T(x) = (\cos 2)(x - 2) + \sin 2$	0.9509	0.9135	0.9093	0.9051	0.8677

6. $f(x) = \csc x$

$f'(x) = -\csc x \cot x$

Tangent line at $(2, \csc 2)$: $y - f(2) = f'(2)(x - 2)$

$y - \csc 2 = (-\csc 2 \cot 2)(x - 2)$

$y = (-\csc 2 \cot 2)(x - 2) + \csc 2$

x	1.9	1.99	2	2.01	2.1
$f(x) = \csc x$	1.0567	1.0948	1.0998	1.1049	1.1585
$T(x) = (-\csc 2 \cot 2)(x - 2) + \csc 2$	1.0494	1.0947	1.0998	1.1048	1.1501

7. $y = f(x) = x^3, f'(x) = 3x^2, x = 1, \Delta x = dx = 0.1$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(1.1) - f(1)$

$= (1.1)^3 - (1)^3$

$= 0.331$

$dy = f'(x)\,dx$

$= f'(1)(0.1)$

$= 3(0.1)$

$= 0.3$

8. $y = f(x) = 1 - 2x^2, f'(x) = -4x, x = 0, \Delta x = dx = -0.1$

$\Delta y = f(x + \Delta x) - f(x)$

$= f(-0.1) - f(0)$

$= [1 - 2(-0.1)^2] - [1 - 2(0)^2] = -0.02$

$dy = f'(x)\,dx$

$= f'(0)(-0.1)$

$= (0)(-0.1) = 0$

9. $y = f(x) = x^4 + 1, f'(x) = 4x^3, x = -1, \Delta x = dx = 0.01$

$\Delta y = f(x + \Delta x) - f(x)$
$= f(-0.99) - f(-1)$
$= [(-0.99)^4 + 1] - [(-1)^4 + 1] \approx -0.0394$

$dy = f'(x)\,dx$
$= f'(-1)(0.01)$
$= (-4)(0.01) = -0.04$

10. $y = f(x) = 2x + 1, f'(x) = 2, x = 2, \Delta x = dx = 0.01$

$\Delta y = f(x + \Delta x) - f(x)$
$= f(2.01) - f(2)$
$= [2(2.01) + 1] - [2(2) + 1] = 0.02$

$dy = f'(x)\,dx$
$= f'(2)(0.01)$
$= 2(0.01) = 0.02$

11. $y = 3x^2 - 4$

$dy = 6x\,dx$

12. $y = 2x^{3/2}$

$dy = 3\sqrt{x}\,dx$

13. $y = \dfrac{x + 1}{2x - 1}$

$dy = \dfrac{-3}{(2x - 1)^2}\,dx$

14. $y = \sqrt{x^2 - 4}$

$dy = \dfrac{x}{\sqrt{x^2 - 4}}\,dx$

15. $y = x\sqrt{1 - x^2}$

$dy = \left(x\dfrac{-x}{\sqrt{1 - x^2}} + \sqrt{1 - x^2}\right)dx = \dfrac{1 - 2x^2}{\sqrt{1 - x^2}}\,dx$

16. $y = \sqrt{x} + \dfrac{1}{\sqrt{x}}$

$dy = \left(\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2x\sqrt{x}}\right)dx = \dfrac{x - 1}{2x\sqrt{x}}\,dx$

17. $y = \dfrac{\sec^2 x}{x^2 + 1}$

$dy = \left[\dfrac{(x^2 + 1)2\sec^2 x \tan x - \sec^2 x(2x)}{(x^2 + 1)^2}\right]dx$

$= \left[\dfrac{2\sec^2 x(x^2 \tan x + \tan x - x)}{(x^2 + 1)^2}\right]dx$

18. $y = x \sin x$

$dy = (x \cos x + \sin x)\,dx$

19. $y = \dfrac{1}{3}\cos\left(\dfrac{6\pi x - 1}{2}\right)$

$dy = -\pi \sin\left(\dfrac{6\pi x - 1}{2}\right)dx$

20. $y = x - \tan^2 x$

$dy = (1 - 2 \tan x \sec^2 x)\,dx$

21. $A = x^2$

$x = 12$

$\Delta x = dx = \pm\frac{1}{64}$

$dA = 2x\,dx$

$\Delta A \approx dA = 2(12)\left(\pm\frac{1}{64}\right)$

$= \pm\frac{3}{8}$ square inches

22. $A = \frac{1}{2}bh, b = 36, h = 50$

$db = dh = \pm 0.25$

$dA = \frac{1}{2}b\,dh + \frac{1}{2}h\,db$

$\Delta A \approx dA = \frac{1}{2}(36)(\pm 0.25) + \frac{1}{2}(50)(\pm 0.25)$

$= \pm 10.75$ square centimeters

23. $A = \pi r^2$

$r = 14$

$\Delta r = dr = \pm\frac{1}{4}$

$\Delta A \approx dA = 2\pi r\,dr = \pi(28)\left(\pm\frac{1}{4}\right)$

$= \pm 7\pi$ square inches

24. $x = 12$ inches

$\Delta x = dx = \pm 0.03$ inch

(a) $V = x^3$

$dV = 3x^2\,dx = 3(12)^2(\pm 0.03)$

$= \pm 12.96$ cubic inches

(b) $S = 6x^2$

$dS = 12x\,dx = 12(12)(\pm 0.03)$

$= \pm 4.32$ square inches

25. (a) $x = 15$ centimeter

$\Delta x = dx = \pm 0.05$ centimeters

$A = x^2$

$dA = 2x\,dx = 2(15)(\pm 0.05)$

$= \pm 1.5$ square centimeters

Percentage error:

$$\frac{dA}{A} = \frac{\pm 1.5}{(15)^2} = 0.00666\ldots = \frac{2}{3}\%$$

(b) $$\frac{dA}{A} = \frac{2x\,dx}{x^2} = \frac{2\,dx}{x} \leq 0.025$$

$$\frac{dx}{x} \leq \frac{0.025}{2} = 0.0125 = 1.25\%$$

26. (a) $C = 56$ centimeters

$\Delta C = dC = \pm 1.2$ centimeters

$$C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$$

$$A = \pi r^2 = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{1}{4\pi}C^2$$

$$dA = \frac{1}{2\pi}C\,dC = \frac{1}{2\pi}(56)(\pm 1.2) = \frac{33.6}{\pi}$$

$$\frac{dA}{A} = \frac{33.6/\pi}{[1/(4\pi)](56)^2} \approx 0.042857 = 4.2857\%$$

(b) $$\frac{dA}{A} = \frac{(1/2\pi)C\,dC}{(1/4\pi)C^2} = \frac{2dC}{C} \leq 0.03$$

$$\frac{dC}{C} \leq \frac{0.03}{2} = 0.015 = 1.5\%$$

27. $r = 6$ inches

$\Delta r = dr = \pm 0.02$ inches

(a) $V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2\,dr = 4\pi(6)^2(\pm 0.02) = \pm 2.88\pi$ cubic inches

(b) $S = 4\pi r^2$

$dS = 8\pi r\,dr = 8\pi(6)(\pm 0.02) = \pm 0.96\pi$ square inches

(c) Relative error: $$\frac{dV}{V} = \frac{4\pi r^2\,dr}{(4/3)\pi r^3} = \frac{3dr}{r}$$

$$= \frac{3}{6}(0.02) = 0.01 = 1\%$$

Relative error: $$\frac{dS}{S} = \frac{8\pi r\,dr}{4\pi r^2} = \frac{2dr}{r}$$

$$= \frac{2(0.02)}{6} = 0.000666\ldots = \frac{2}{3}\%$$

28. $P = (500x - x^2) - \left(\frac{1}{2}x^2 - 77x + 3000\right)$, x changes from 115 to 120

$dP = (500 - 2x - x + 77)dx = (577 - 3x)\,dx = [577 - 3(115)](120 - 115) = 1160$

Approximate percentage change: $\frac{dP}{P}(100) = \frac{1160}{43517.50}(100) \approx 2.7\%$

29. $V = \pi r^2 h = 40\pi r^2$, $r = 5$ cm, $h = 40$ cm, $dr = 0.2$ cm

$\Delta V \approx dV = 80\pi r\,dr = 80\pi(5)(0.2) = 80\pi$ cm^3

30. $V = \frac{4}{3}\pi r^3$, $r = 100$ cm, $dr = 0.2$ cm

$\Delta V \approx dV = 4\pi r^2\,dr = 4\pi(100)^2(0.2) = 8000\pi$ cm^3

31. (a) $T = 2\pi\sqrt{L/g}$

$$dT = \frac{\pi}{g\sqrt{L/g}}\,dL$$

Relative error:

$$\frac{dT}{T} = \frac{(\pi\,dL)/(g\sqrt{L/g})}{2\pi\sqrt{L/g}}$$

$$= \frac{dL}{2L}$$

$$= \frac{1}{2}\,(\text{relative error in } L)$$

$$= \frac{1}{2}(0.005) = 0.0025$$

Percentage error: $\frac{dT}{T}(100) = 0.25\% = \frac{1}{4}\%$

(b) $(0.0025)(3600)(24) = 216$ seconds

$= 3.6$ minutes

32. $T = 2\pi\sqrt{L/g}$

$$T^2 = 4\pi^2\left(\frac{L}{g}\right)$$

$$g = \frac{4\pi^2 L}{T^2}$$

$$dg = 4\pi^2 L\,\frac{(-2)}{T^3}\,dT$$

Relative error in g:

$$\frac{dg}{g} = \frac{(-8\pi^2 L\,dT)/T^3}{(4\pi^2 L)/T^2}$$

$$= -2\frac{dT}{T} = -2\,(\text{relative error in period})$$

$$= -2(0.001) = -0.002$$

Percentage error in g: $\left|\frac{dg}{g}\right|(100) = \frac{1}{5}\%$

33. $E = IR$

$$R = \frac{E}{I}$$

$$dR = -\frac{E}{I^2}\,dI$$

$$\frac{dR}{R} = \frac{-(E/I^2)\,dI}{E/I} = -\frac{dI}{I}$$

$$\left|\frac{dR}{R}\right| = \left|-\frac{dI}{I}\right| = \left|\frac{dI}{I}\right|$$

34. $C = \dfrac{80{,}000p}{100 - p},\ 0 \le p < 100$

(a) $p = 40$

$$\Delta p = dp = 2$$

$$dC = \frac{8{,}000{,}000}{(100 - p)^2}\,dp$$

$$= \frac{8{,}000{,}000}{(100 - 40)^2}(2) \approx \$4444.44$$

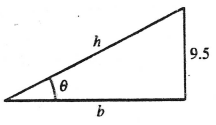
(b) $p = 75$

$$\Delta p = dp = 2$$

$$dC = \frac{8{,}000{,}000}{(100 - 75)^2}(2) = \$25{,}600.00$$

35. $\theta = 26°45' = 26.75°$

$d\theta = \pm 15' = \pm 0.25°$



(a) $h = 9.5 \csc\theta$

$$dh = -9.5 \csc\theta \cot\theta\,d\theta$$

$$\frac{dh}{h} = -\cot\theta\,d\theta$$

$$\left|\frac{dh}{h}\right| = (\cot 26.75°)(0.25°)$$

Converting to radians, $(\cot 0.4669)(0.0044)$
$\approx 0.0087 = 0.87\%$ (in radians).

(b) $\left|\dfrac{dh}{h}\right| = \cot\theta\,d\theta \le 0.02$

$$\frac{d\theta}{\theta} \le \frac{0.02}{\theta(\cot\theta)} = \frac{0.02\tan\theta}{\theta}$$

$$\frac{d\theta}{\theta} \le \frac{0.02\tan 26.75°}{26.75°} \approx \frac{0.02\tan 0.4669}{0.4669}$$

$\approx 0.0216 = 2.16\%$ (in radians)