AP Calculus   Applications of Using the Definite Integral for Accumulation   Critical HW

1) **Calculator Allowed**

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

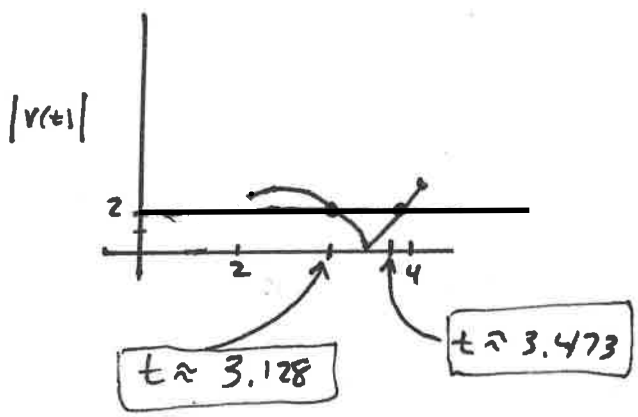
(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) speed $= |v(t)|$



b) $s(t) = s(0) + \int_0^t v(t)\,dt$

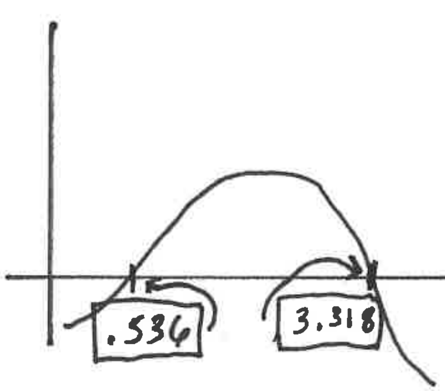
$s(t) = 10 + \int_0^t v(t)\,dt$

$s(5) = 10 + \int_0^5 v(t)\,dt$

$= 10 - 19.207$

$= \boxed{-9.207}$

c) where $v(t)$ crosses the x-axis (changes sign)



d) $v(4) = -11.476$

$a(4) = -22.296$

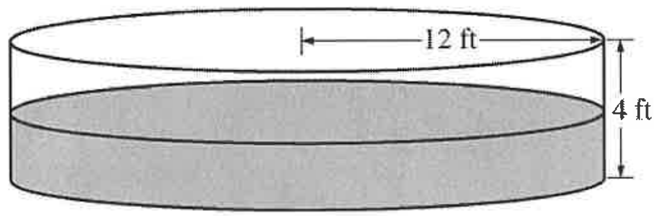
by finding $v'(4)$ on calculator

$\boxed{\text{Speed is increasing}}$

as both velocity & acceleration are Negative (same sign)

2)

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



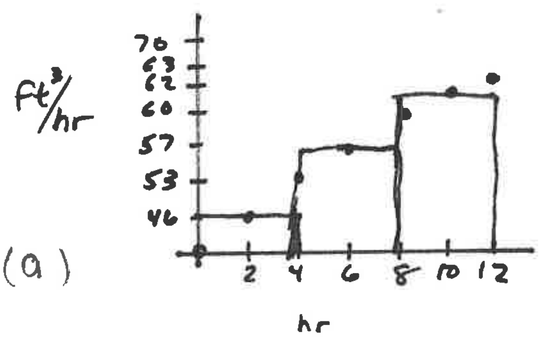
The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(a)



$4 \cdot 46 + 4 \cdot 57 + 4 \cdot 62$

$\boxed{660 \text{ ft}^3}$

(b) $\int_0^{12} R(t)\,dt$

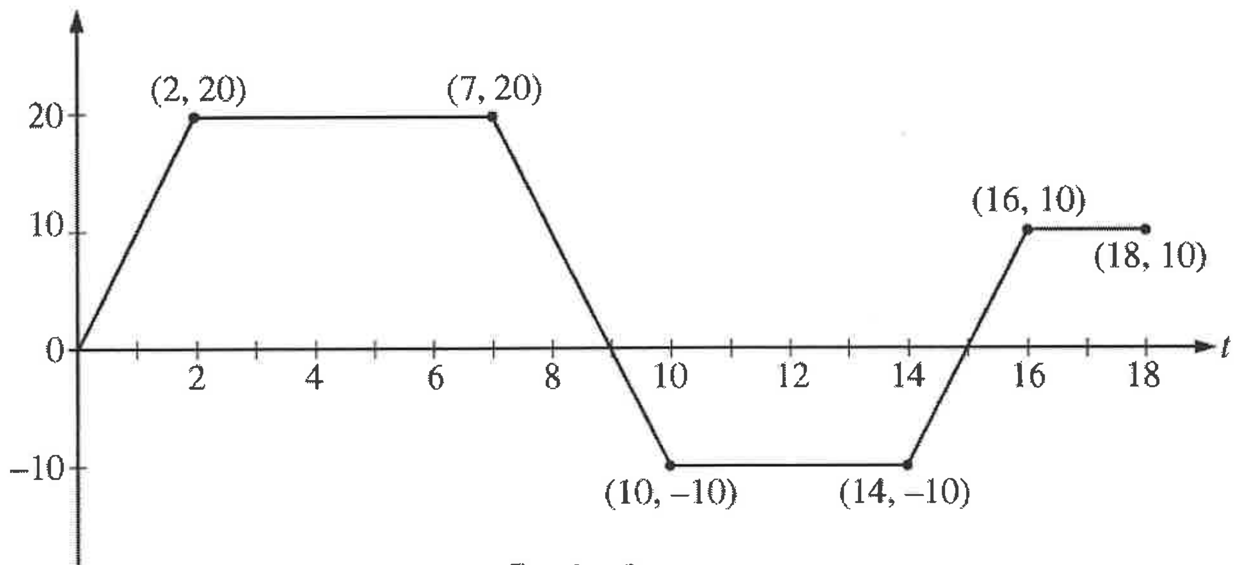
$\boxed{\approx 225.594 \text{ ft}^3}$

(c) $V = \underset{\text{amount at } t=0}{1000} + \underset{\text{amount added from } [0,12]}{660} - \underset{\text{amount leaked out from } [0,12]}{225.594}$

1434.406 so $\boxed{1434 \text{ ft}^3}$

3)

No calculator is allowed for these problems.



Graph of v

A squirrel starts at building A at time $t = 0$ and travels along a straight, horizontal wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A ? How far from building A is the squirrel at that time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(a) where velocity changes sign (so... when v graph crosses x-axis)

$\boxed{t = 9 \ \& \ t = 15}$

(b) $\int_0^9 v(t)\,dt = \frac{(9+5)\cdot 20}{2}$ $= 140$ ft away from the building

then $\int_9^{15} v(t)\,dt = -\frac{(6+4)\cdot 10}{2}$ $= -50$ 50 ft back towards the building

then $\int_{15}^{18} v(t)\,dt = \frac{(3+2)\cdot 10}{2}$ $= 25$ 25 ft away from building again

$\boxed{t = 9 \text{ squirrel is } 140 \text{ ft from building}}$

(c) DISTANCE: 140 ft + 50 ft + 25 ft

$\boxed{215 \text{ ft travelled total}}$

Note: This is not its position at $t = 18$! That would be 115 ft from the building

4) AP MULTIPLE CHOICE EXAMPLES

1) A particle moves in a straight line with velocity $v(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 2$?

(A) $\frac{1}{3}$ (B) $\frac{7}{3}$ (C) 3 (D) 7 (E) 8

$$\int_1^2 v(t)\,dt = \int_1^2 t^2\,dt$$

$$= \frac{t^3}{3}\Big|_1^2 = \frac{8}{3} - \frac{1}{3} = \boxed{7/3}$$

2) At $t = 0$ a particle starts at rest and moves along a line in such a way that at time t its acceleration is $24t^2$ feet per second per second. Through how many feet does the particle move during the first 2 seconds?

$v(0) = 0$

(A) 32 (B) 48 (C) 64 (D) 96 (E) ~~192~~

"FIRST "2" seconds"

$a(t) = 24t^2$

$\int a(t)\,dt = 24\frac{t^3}{3}$

$v(t) = 8t^3 + C$

$0 = 8(0)^3 + C$

$0 = C$

$v(t) = 8t^3$

$$\int_0^2 v(t)\,dt = 8 \cdot \frac{t^4}{4}\Big|_0^2 = 2t^4\Big|_0^2 = 2(16) - 2(0) = 32$$

3) **Graphing Calculator Allowed**

The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $\boxed{7,500e^2}$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

$$7500 + \int_0^5 3000e^{(2t/5)}\,dt$$

$7500 + 47,917.92$

$\approx 55,417.921$

4) A point moves in a straight line so that its distance at time t from a fixed point of the line is $8t - 3t^2$. What is the *total* distance covered by the point between $t = 1$ and $t = 2$?

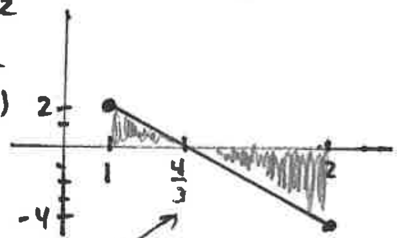
(A) 1 (B) $\frac{4}{3}$ (C) $\frac{5}{3}$ (D) 2 (E) 5

$S(t) = 8t - 3t^2$

$v(t) = 8 - 6t$

$0 = 8 - 6t$

$\frac{4}{3} = t$



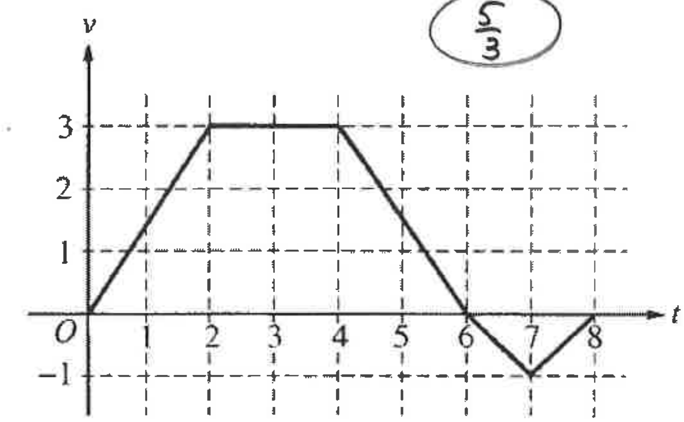
$\int_1^{4/3} v(t)\,dt + \left|\int_{4/3}^{2} v(t)\,dt\right|$

$\frac{\frac{1}{3}\cdot 2}{2} + \left|-\frac{\frac{2}{3}\cdot 4}{2}\right|$

$\frac{1}{3} + \frac{4}{3}$

$\frac{5}{3}$

5)



A bug begins to crawl up a vertical wire at time $t = 0$. The velocity v of the bug at time t, $0 \le t \le 8$, is given by the function whose graph is shown above.

What is the total distance the bug traveled from $t = 0$ to $t = 8$?

(A) 14 (B) 13 (C) 11 (D) 8 (E) 6

$\int_0^8 |v(t)\,dt| = \int_0^6 v(t)\,dt + \int_6^8 |v(t)|\,dt$

$= \frac{(6+2)\cdot 3}{2} + \frac{2\cdot 1}{2} \longrightarrow 12 + 1 = 13$ ↖ TOTAL

up down

6) **Graphing Calculator Allowed**

A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 7$. If the initial velocity of the particle is 6, at what time t during the interval $0 \le t \le 4$ is the particle farthest to the right?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Initial Velocity

$V(0) = 6$

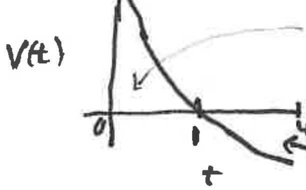
$a(t) = 2t - 7$

$v(t) = \frac{2t^2}{2} - 7t + c$

$6 = 0 - 0 + c$

$6 = c$

$V(t) = t^2 - 7t + 6$



$\int_0^1 v(t)\,dt$ = distance travelled RIGHT

distance travelled left

7) Graphing Calculator Allowed

Let g be a differentiable function such that $g(4) = 0.325$ and $g'(x) = \frac{1}{x}e^{-x}\left(\cos\left(\frac{x}{100}\right)\right)$. What is the value of $g(1)$?

$x=4$ $x=1$

- (A) 0.109
- (B) 0.216
- (C) 0.541
- (D) 0.688

$$g(1) + \int_1^4 g'(x)\,dx = g(4)$$

$$g(1) = g(4) - \int_1^4 g'(x)\,dx$$

$$= .325 - .216$$

$$= .109$$