Average & Instantaneous Rate of Change

w-up: Graph $y = \frac{1}{4}x^2$

Average Rate of Change VS. Instantaneous Rate of Change

How do we find the average for a set of data?

Average Rate of Change: Average change over an INTERVAL which is the slope of the *secant* line.

EX 1) Find the average ROC for $f(x) = \frac{1}{4}x^2$ for the given intervals.

A) [1, 4] B) [1, 2] C) [1, 1.01]

Instantaneous Rate of Change: the change at any moment which is the slope of the tangent line AT a point

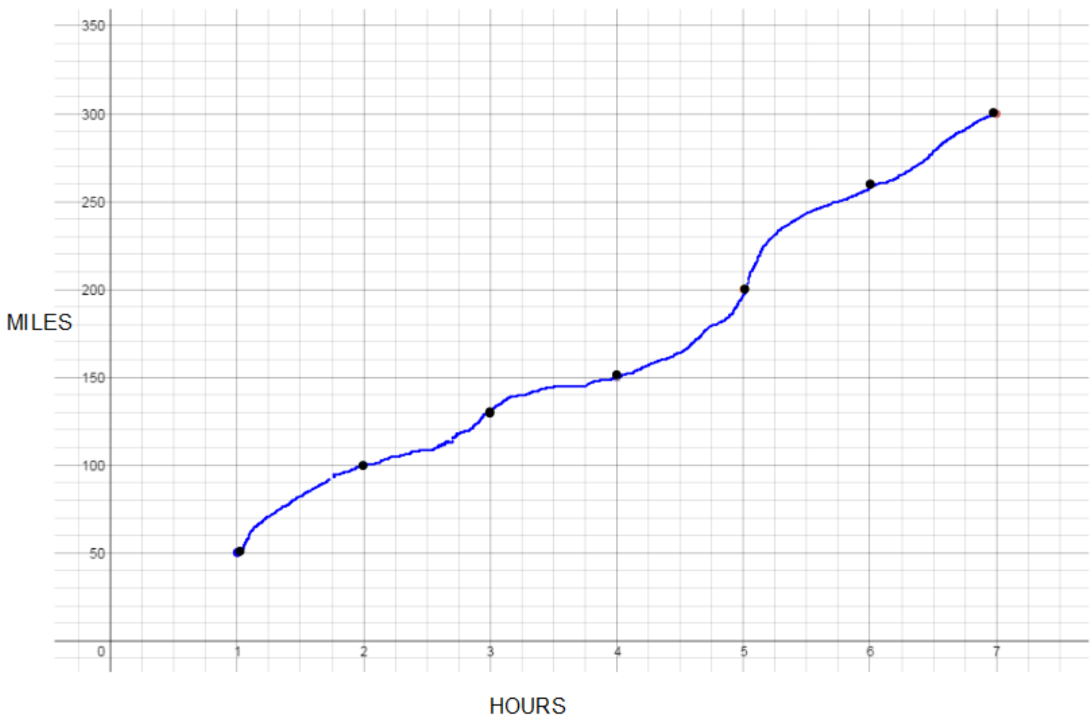
EX 2) Find the instantaneous ROC for $f(x) = \frac{1}{4}x^2$ at x = 1.

Notice when using an interval where Δx is extremely small("C" from example #1) the average rate of change is close to the ________________________

Linear Approximation: Using the average ROC over a very small interval to approximate the instantaneous ROC

AP Application of Linear Approximation

The following graph represents the distance a car has traveled after a certain time(in hours)



Find the average ROC **from** hour 3 to hour 4. Label your answer. What have you found?

Estimate the instantaneous ROC **at** hour 4. Label your answer. What have you found?

Note: Finding slope(derivative) on a distance vs. time graph yields **velocity**(speed in this case)

Rectilinear Motion

Rectilinear Motion: motion of an object moving in a straight line.

The ***derivative*** of any POSITION function is instantaneous velocity.

The ***derivative*** of any VELOCITY function is instantaneous acceleration.

NOTE: Because slope can be negative, both velocity and acceleration can be a negative value. This simply implies DIRECTION of the movement.

Positive velocity/acceleration means movement RIGHT or UPWARD

Negative velocity/acceleration means movement LEFT or DOWNWARD

AP Example

A jogger runs along a straight track. The jogger's position is given by the function $p(t)$, where t is measured in minutes since the start of the run. During the first minute of the run, the jogger's acceleration is proportional to the square root of the time since the start of the run. Which of the following is a differential equation that describes this relationship, where k is a positive constant?

(A) $\frac{dp}{dt} = k\sqrt{t}$

(B) $\frac{dp}{dt} = k\sqrt{p}$

(C) $\frac{d^2p}{dt^2} = k\sqrt{t}$

(D) $\frac{d^2p}{dt^2} = k\sqrt{p}$

SPEED: the magnitude(quantity) of velocity so Speed = |Velocity|

It is very important you read AP questions carefully and note whether velocity or speed is being given/asked for!!!!!

Using the Classic Projectile Formula for Objects Thrown Upward

$$H(t) = -16t^2 + V_o t + h_o$$

represents the Height(in feet) of an object *H(t)* after time *t*(in seconds) when thrown straight into the air where V_o represents original velocity and h_o represents original height.

So, the distance(height) over time function for an object thrown straight into the air with velocity of 88 ft/sec from 25 feet is $H(t) = -16t^2 + 88t + 25$. This can now find the height of this object after time *t*.

EX) What is the height when t = 2?

EX) When is the object 50 feet above the ground?

EX) What is the velocity of the object when it is 50 feet above the ground?

EX) What is the acceleration of the object when it is 50 feet above the ground?

AP EXAMPLE

USE THE GRAPHING CALCULATOR TO ANSWER THE FOLLOWING QUESTION

The height *h* in meters, of an object at time *t* is given by $h(t) = 24t + 24t^{3/2} - 16t^2$. What is the height of the object at the instant when it reaches its maximum velocity?