Find the average rate of change of the function over the given interval. Compare this average rate of change with the instantaneous rates of change at the endpoints of the interval.

1a) $f(t) = t^2 - 7, \quad [3, 3.1]$

$$\frac{f(3.1)-f(3)}{3.1-3}$$

Avg. ROC = 6.1

$f'(x) = 2x$

$f'(3.1) = 6.2$

$f'(3) = 6$

1b) $f(t) = 4t + 5, \quad [1, 2]$

$(1, 9) \ (2, 13)$

$\frac{13-9}{2-1} = 4$

AVG ROC = 4

$f'(t) = 4$

$f'(1) = 4$

$f'(2) = 4$

1c) $f(x) = \frac{-1}{x}, \quad [1, 2]$

$(1, -1) \ (2, -\frac{1}{2})$

$\frac{-\frac{1}{2} - (-1)}{2-1} = \frac{\frac{1}{2}}{1}$

AVG ROC = 1/2

$f(t) = -x^{-1}$ $\quad f'(t) = x^{-2}$

$f'(t) = \frac{1}{x^2}$

$f'(1) = 1$

$f'(2) = 1/4$

2a) **Use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects to answer the following questions.**

A silver dollar is dropped from the top of a building that is 1362 feet tall.

(a) Determine the position and velocity functions for the coin.

(b) Determine the average velocity on the interval [1, 2].

(c) Find the instantaneous velocities when $t = 1$ and $t = 2$.

(d) Find the time required for the coin to reach ground level.

(e) Find the velocity of the coin at impact.

(a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) $\frac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

(c) $v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec

When $t = 2$: $v(2) = -64$ ft/sec

(d) $-16t^2 + 1362 = 0$

$$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

(e) $v\left(\frac{\sqrt{1362}}{4}\right) = -32\left(\frac{\sqrt{1362}}{4}\right)$

$= -8\sqrt{1362} \approx -295.242$ ft/sec

2b) A ball is thrown straight down from the top of a 220-foot building with an initial velocity of (-22) feet per second.

$H(t) = -16t^2 - 22t + 220$

What is its velocity after 3 seconds?

$V(t) = -32t - 22$

$V(3) = -118$ ft/sec

What is its velocity after falling 108 feet?

So, at what "t" is object at 112 ft? start - fell 108 ft
220 - 108

$112 = -16t^2 - 22t + 220$

$0 = -16t^2 - 22t + 108$

$t = 2$ or $\cancel{-\frac{27}{8}}$

use G.C. or

$V(2) = -86$ ft/sec

3a)

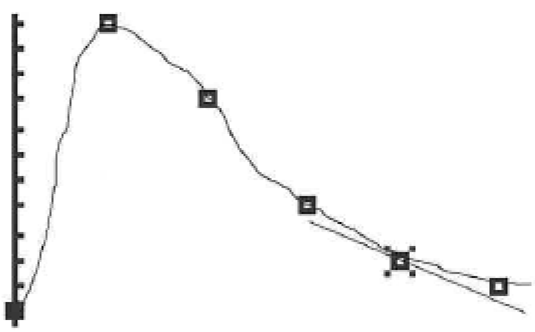
t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

(a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

(b) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

(a)



$W'(12)$ means slope of the function when t = 12 days. We can estimate that using average rate of change(slope between) over [9, 12] which is $-2/3\ ^{\circ}C/day$

Note: average roc over [12, 15] ($-1/3\ ^{\circ}C/day$)

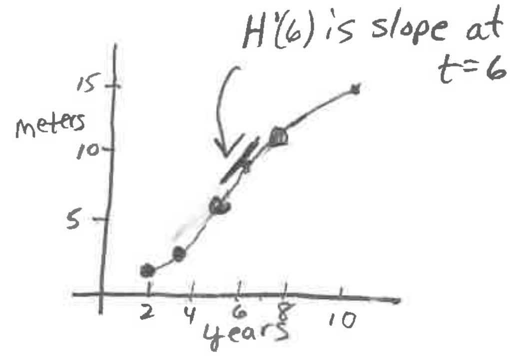
Would also be an appropriate answer!

(b) Using a calculator, $\frac{d}{dX}(20+10Xe^{-X/3})|_{X=12}$
-.54946917

So, on the 12th day the water is cooling $-.549\ ^{\circ}C/day$

3b)

t (years)	2	3	5	7	10
$H(t)$ (meters)	1.5	2	6	11	15



The height of a tree at time t is given by a twice-differentiable function H, where $H(t)$ is measured in meters and t is measured in years. Selected values of $H(t)$ are given in the table above.

(a) Use the data in the table to estimate $H'(6)$. Using correct units, interpret the meaning of $H'(6)$ in the context of the problem.

use $(5,6)$ & $(7,11)$

$H'(6) \approx \frac{11-6}{7-5}\frac{m}{y} \approx \frac{5}{2}$ m/yr

yearly INCREASE in height of tree in 6th year

(b) If the function $H(t) = -.057t^3 + 1.014t^2 - 3.378t + 4.623$ is a good representation for this data, use your graphing calculator to check your estimate from part (a).

$H'(6) \approx 2.634$ m/yr

4) AP MULTIPLE CHOICE EXAMPLES

1) A particle moves along the x-axis so that at any time $t \geq 0$ its position is given by $x(t) = t^3 - 3t^2 - 9t + 1$. For what values of t is the particle at rest? ← velocity is ZERO

(A) No values (B) 1 only (C) 3 only (D) 5 only (E) 1 and 3

$v(t) = 3t^2 - 6t - 9$

$0 = 3t^2 - 6t - 9$

$0 = 3(t^2 - 2t - 3)$

$0 = 3(t-3)(t+1)$

$\cancel{3 \neq 0}$ $t-3=0$ $t+1=0$

$\boxed{t=3}$ $\cancel{t=-1}$

2) The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0 $\boxed{\text{(B) 2}}$ (C) 4 (D) 8 (E) 12

$v(t) = 2t + 4$

$a(t) = 2$

$a(4) = 2$

3) If the position of a particle on the x-axis at time t is $-5t^2$, then the average velocity of the particle for $0 \le t \le 3$ is

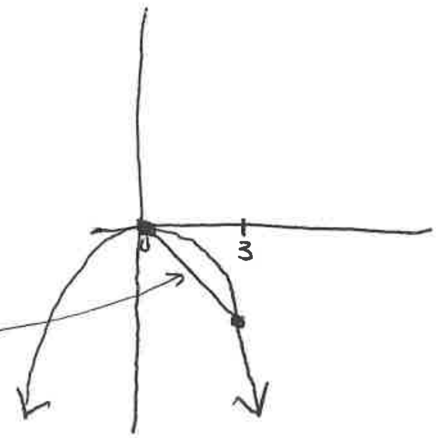
slope across position function from $x=0$ to $x=3$

(A) -45 (B) -30 (C) -15 (D) -10 (E) -5

$s(t) = -5t^2$

$(0, 0)$
$(3, -45)$

$$\frac{-45-0}{3-0} = \frac{-45}{3} = -15$$



4) GRAPHING CALCULATOR NEEDED

A particle moves along a line so that at time t, where $0 \le t \le \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity of the particle when its acceleration is zero?

(A) -5.19 (B) 0.74 (C) 1.32 (D) 2.55 (E) 8.13

$v(t) = 4\sin t - t$

$v(1.318) \approx 2.555$

$a(t) = 4\cos t - 1$

use G.C. (RADIAN MODE!!!) $\rightarrow$ $0 = 4\cos t - 1$

$t \approx 1.318$