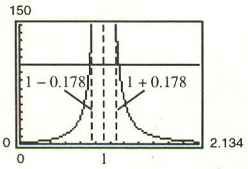
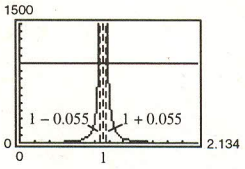
63. $f(x) = \dfrac{x+2}{(x-1)^2}$

(a) $\delta \approx 0.178$ for $M = 100$.



(b) $\delta \approx 0.055$ for $M = 1000$.



(c) As M increases, δ decreases.

Review Exercises for Chapter 1

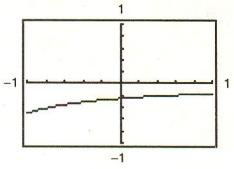
1. Calculus required. Using a graphing utility, you can estimate the length to be 8.3. Or, the length is slightly longer than the distance between the two points, 8.25.

2. Precalculus. $L = \sqrt{(9-1)^2 + (3-1)^2} \approx 8.25$

3.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.26	-0.25	-0.250	-0.2499	-0.249	-0.24

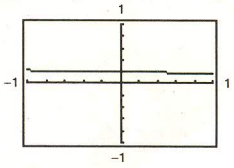
$\lim\limits_{x\to 0} f(x) \approx -0.25$



4.

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	0.202	0.2	0.2	0.2	0.2	0.198

$\lim\limits_{x\to 0} f(x) \approx 0.2$



5. $h(x) = \dfrac{x^2 - 2x}{x}$

(a) $\lim\limits_{x\to 0} h(x) = -2$

(b) $\lim\limits_{x\to -1} h(x) = -3$

6. $g(x) = \dfrac{3x}{x-2}$

(a) $\lim\limits_{x\to 2^+} g(x) = \infty$

(b) $\lim\limits_{x\to 0} g(x) = 0$

7. $\lim\limits_{x\to 2} (5x - 3) = 5(2) - 3 = 7$

8. $\lim\limits_{x\to 2} (3x + 5) = 3(2) + 5 = 11$

9. $\lim\limits_{x\to 2} (5x - 3)(3x + 5) = [5(2) - 3][3(2) + 5]$
$= 7 \cdot 11 = 77$

10. $\lim\limits_{x\to 2} \left(\dfrac{3x+5}{5x-3}\right) = \dfrac{3(2)+5}{5(2)-3} = \dfrac{11}{7}$

11. $\lim\limits_{t\to 3} \dfrac{t^2+1}{t} = \dfrac{3^2+1}{3} = \dfrac{10}{3}$

12. $\lim\limits_{t\to 3} \dfrac{t^2-9}{t-3} = \lim\limits_{t\to 3} (t+3) = 6$

13. $\lim\limits_{t\to -2} \dfrac{t+2}{t^2-4} = \lim\limits_{t\to -2} \dfrac{1}{t-2} = -\dfrac{1}{4}$

14. $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x} = \lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{x} \cdot \dfrac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$
$= \lim\limits_{x\to 0} \dfrac{1}{\sqrt{4+x}+2} = \dfrac{1}{4}$

15. $\lim\limits_{x\to 0} \dfrac{[1/(x+1)]-1}{x} = \lim\limits_{x\to 0} \dfrac{1-(x+1)}{x(x+1)}$
$= \lim\limits_{x\to 0} \dfrac{-1}{x+1} = -1$

16. $$\lim_{s\to 0}\frac{(1/\sqrt{1+s})-1}{s}=\lim_{s\to 0}\left[\frac{(1/\sqrt{1+s})-1}{s}\cdot\frac{(1/\sqrt{1+s})+1}{(1/\sqrt{1+s})+1}\right]$$
$$=\lim_{s\to 0}\frac{[1/(1+s)]-1}{s\left[(1/\sqrt{1+s})+1\right]}=\lim_{s\to 0}\frac{-1}{(1+s)\left[(1/\sqrt{1+s})+1\right]}=-\frac{1}{2}$$

17. $$\lim_{x\to -5}\frac{x^3+125}{x+5}=\lim_{x\to -5}\frac{(x+5)(x^2-5x+25)}{x+5}$$
$$=\lim_{x\to -5}(x^2-5x+25)$$
$$=75$$

18. $$\lim_{x\to -2}\frac{x^2-4}{x^3+8}=\lim_{x\to -2}\frac{(x+2)(x-2)}{(x+2)(x^2-2x+4)}$$
$$=\lim_{x\to -2}\frac{x-2}{x^2-2x+4}$$
$$=-\frac{4}{12}=-\frac{1}{3}$$

19. $$\lim_{x\to 0^+}\left(x-\frac{1}{x^3}\right)=-\infty$$

20. $$\lim_{x\to 2^+}\frac{1}{\sqrt[3]{x^2-4}}=\infty$$
$$\lim_{x\to 2^-}\frac{1}{\sqrt[3]{x^2-4}}=-\infty$$
Thus, $\lim_{x\to 2}\frac{1}{\sqrt[3]{x^2-4}}$ does not exist.

21. $$\lim_{\Delta x\to 0}\frac{\sin[(\pi/6)+\Delta x]-(1/2)}{\Delta x}=\lim_{\Delta x\to 0}\frac{\sin(\pi/6)\cos\Delta x+\cos(\pi/6)\sin\Delta x-(1/2)}{\Delta x}$$
$$=\lim_{\Delta x\to 0}\frac{1}{2}\cdot\frac{(\cos\Delta x-1)}{\Delta x}+\lim_{\Delta x\to 0}\frac{\sqrt{3}}{2}\cdot\frac{\sin\Delta x}{\Delta x}$$
$$=0+\frac{\sqrt{3}}{2}(1)=\frac{\sqrt{3}}{2}$$

22. $$\lim_{\Delta x\to 0}\frac{\cos(\pi+\Delta x)+1}{\Delta x}=\lim_{\Delta x\to 0}\frac{\cos\pi\cos\Delta x-\sin\pi\sin\Delta x+1}{\Delta x}$$
$$=\lim_{\Delta x\to 0}\left[-\frac{(\cos\Delta x-1)}{\Delta x}\right]-\lim_{\Delta x\to 0}\left[\sin\pi\frac{\sin\Delta x}{\Delta x}\right]$$
$$=-0-(0)(1)=0$$

23. $$\lim_{x\to -2^-}\frac{2x^2+x+1}{x+2}=-\infty$$

24. $$\lim_{x\to (1/2)^+}\frac{x}{2x-1}=\infty$$

25. $$\lim_{x\to -1^+}\frac{x+1}{x^3+1}=\lim_{x\to -1^+}\frac{1}{x^2-x+1}=\frac{1}{3}$$

26. $$\lim_{x\to -1^-}\frac{x+1}{x^4-1}=\lim_{x\to -1^-}\frac{1}{(x^2+1)(x-1)}=-\frac{1}{4}$$

27. $$\lim_{x\to 1^-}\frac{x^2+2x+1}{x-1}=-\infty$$

28. $$\lim_{x\to -1^+}\frac{x^2-2x+1}{x+1}=\infty$$

29. $$\lim_{x\to 0^+}\frac{\sin 4x}{5x}=\lim_{x\to 0^+}\left[\frac{4}{5}\left(\frac{\sin 4x}{4x}\right)\right]=\frac{4}{5}$$

30. $$\lim_{x\to 0^+}\frac{\sec x}{x}=\infty$$

31. $$\lim_{x\to 0^+}\frac{\csc 2x}{x}=\lim_{x\to 0^+}\frac{1}{x\sin 2x}=\infty$$

32. $$\lim_{x\to 0^-}\frac{\cos^2 x}{x}=-\infty$$

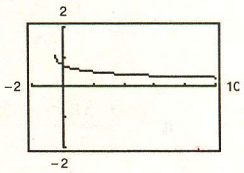
33. $f(x) = \dfrac{\sqrt{2x+1} - \sqrt{3}}{x-1}$

(a)

x	1.1	1.01	1.001	1.0001
$f(x)$	0.5680	0.5764	0.5773	0.5773

$$\lim_{x\to 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} \approx 0.577 \quad \left(\text{Actual limit is } \sqrt{3}/3.\right)$$

(b)



(c)
$$\lim_{x\to 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} = \lim_{x\to 1^+} \frac{\sqrt{2x+1} - \sqrt{3}}{x-1} \cdot \frac{\sqrt{2x+1} + \sqrt{3}}{\sqrt{2x+1} + \sqrt{3}}$$
$$= \lim_{x\to 1^+} \frac{(2x+1) - 3}{(x-1)\left(\sqrt{2x+1} + \sqrt{3}\right)}$$
$$= \lim_{x\to 1^+} \frac{2}{\sqrt{2x+1} + \sqrt{3}}$$
$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

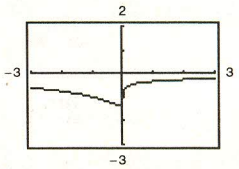
34. $f(x) = \dfrac{1 - \sqrt[3]{x}}{x-1}$

(a)

x	1.1	1.01	1.001	1.0001
$f(x)$	−0.3228	−0.3322	−0.3332	−0.3333

$$\lim_{x\to 1^+} \frac{1 - \sqrt[3]{x}}{x-1} \approx -0.333 \quad \left(\text{Actual limit is } -\tfrac{1}{3}.\right)$$

(b)



(c)
$$\lim_{x\to 1^+} \frac{1 - \sqrt[3]{x}}{x-1} = \lim_{x\to 1^+} \frac{1 - \sqrt[3]{x}}{x-1} \cdot \frac{1 + \sqrt[3]{x} + \left(\sqrt[3]{x}\right)^2}{1 + \sqrt[3]{x} + \left(\sqrt[3]{x}\right)^2}$$
$$= \lim_{x\to 1^+} \frac{1 - x}{(x-1)\left[1 + \sqrt[3]{x} + \left(\sqrt[3]{x}\right)^2\right]}$$
$$= \lim_{x\to 1^+} \frac{-1}{1 + \sqrt[3]{x} + \left(\sqrt[3]{x}\right)^2}$$
$$= -\frac{1}{3}$$

35. $f(x) = [\![x + 3]\!]$

$\lim\limits_{x\to k^+} [\![x + 3]\!] = k + 3$ where k is an integer.

$\lim\limits_{x\to k^-} [\![x + 3]\!] = k + 2$ where k is an integer.

Nonremovable discontinuity at each integer k

Continuous on $(k, k+1)$ for all integers k

36. $f(x) = \dfrac{3x^2 - x - 2}{x-1} = \dfrac{(3x+2)(x-1)}{x-1}$

$\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} (3x + 2) = 5$

Removable discontinuity at $x = 1$

Continuous on $(-\infty, 1) \cup (1, \infty)$

37. $f(x) = \begin{cases} \dfrac{3x^2 - x - 2}{x-1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{3x^2 - x - 2}{x-1}$$
$$= \lim_{x\to 1} (3x + 2) = 5 \neq 0$$

Removable discontinuity at $x = 1$

Continuous on $(-\infty, 1) \cup (1, \infty)$

38. $f(x) = \begin{cases} 5 - x, & x \leq 2 \\ 2x - 3, & x > 2 \end{cases}$

$\lim\limits_{x\to 2^-} (5 - x) = 3$

$\lim\limits_{x\to 2^+} (2x - 3) = 1$

Nonremovable discontinuity at $x = 2$

Continuous on $(-\infty, 2) \cup (2, \infty)$

39. $f(x) = \dfrac{1}{(x-2)^2}$

$$\lim_{x\to 2} \frac{1}{(x-2)^2} = \infty$$

Nonremovable discontinuity at $x = 2$
Continuous on $(-\infty, 2) \cup (2, \infty)$

40. $f(x) = \sqrt{\dfrac{x+1}{x}} = \sqrt{1 + \dfrac{1}{x}}$

$$\lim_{x\to 0^+} \sqrt{1 + \frac{1}{x}} = \infty$$

Domain: $(-\infty, -1], (0, \infty)$

Nonremovable discontinuity at $x = 0$
Continuous on $(-\infty, -1] \cup (0, \infty)$

41. $f(x) = \dfrac{3}{x+1}$

$$\lim_{x\to 1^-} f(x) = -\infty$$

$$\lim_{x\to 1^+} f(x) = \infty$$

Nonremovable discontinuity at $x = -1$
Continuous on $(-\infty, -1) \cup (-1, \infty)$

42. $f(x) = \dfrac{x+1}{2x+2}$

$$\lim_{x\to -1} \frac{x+1}{2(x+1)} = \frac{1}{2}$$

Removable discontinuity at $x = -1$
Continuous on $(-\infty, -1) \cup (-1, \infty)$

43. $f(x) = \csc \dfrac{\pi x}{2}$

Nonremovable discontinuities at each even integer. Continuous on

$$(2k, 2k + 2)$$

for all integers k.

44. $f(x) = \tan 2x$

Nonremovable discontinuities when

$$x = \frac{(2n+1)\pi}{4}$$

Continuous on

$$\left(\frac{(2n-1)\pi}{4}, \frac{(2n+1)\pi}{4}\right)$$

for all integers n.

45. $f(2) = 5$

Find c so that $\lim_{x\to 2^+} (cx + 6) = 5$.

$$c(2) + 6 = 5$$

$$2c = -1$$

$$c = -\frac{1}{2}$$

46. $\lim_{x\to 1^+} (x + 1) = 2$

$\lim_{x\to 3^-} (x + 1) = 4$

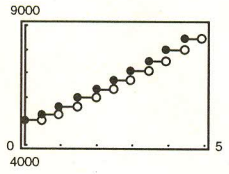
Find b and c so that $\lim_{x\to 1^-} (x^2 + bx + c) = 2$ and $\lim_{x\to 3^+} (x^2 + bx + c) = 4$.

Consequently we get $\quad 1 + b + c = 2 \quad$ and $\quad 9 + 3b + c = 4$.

Solving simultaneously, $\quad b = -3 \quad$ and $\quad c = 4$.

47. $A = 5000(1.06)^{[\![2t]\!]}$

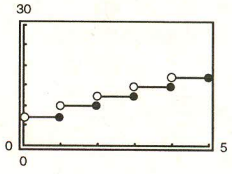
Nonremovable discontinuity every 6 months



48. $C = 9.80 + 2.50[-[\![-x]\!] - 1], \; x > 0$

$\quad = 9.80 - 2.50[[\![-x]\!] + 1]$

C has a nonremovable discontinuity at each integer.



49. $g(x) = 1 + \dfrac{2}{x}$

Vertical asymptote at $x = 0$

50. $h(x) = \dfrac{4x}{4 - x^2}$

Vertical asymptotes at $x = 2$ and $x = -2$

51. $f(x) = \dfrac{8}{(x - 10)^2}$

Vertical asymptote at $x = 10$

52. $f(x) = \csc \pi x$

Vertical asymptote at every integer k

53. $C = \dfrac{80{,}000p}{100 - p}$, $0 \le p < 100$

(a) $C(15) \approx \$14{,}117.65$ (b) $C(50) = \$80{,}000$

(c) $C(90) = \$720{,}000$ (d) $\displaystyle\lim_{p \to 100^-} \frac{80{,}000p}{100 - p} = \infty$

54. $f(x) = \dfrac{\tan 2x}{x}$

(a)

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	2.0271	2.0003	2.0000	2.0000	2.0003	2.0271

$$\lim_{x \to 0} \frac{\tan 2x}{x} = 2$$

(b) Yes, define

$$f(x) = \begin{cases} \dfrac{\tan 2x}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}.$$

Now $f(x)$ is continuous at $x = 0$.

55.
$$\lim_{t \to a} \frac{s(a) - s(t)}{a - t} = \lim_{t \to 4} \frac{(-4.9(4)^2 + 200) - (-4.9t^2 + 200)}{4 - t}$$
$$= \lim_{t \to 4} \frac{4.9(t - 4)(t + 4)}{4 - t}$$
$$= \lim_{t \to 4} -4.9(t + 4) = -39.2 \text{ m/sec}$$

56. $s(t) = 0 \implies -4.9t^2 + 200 = 0 \implies t^2 \approx 40.816 \implies t \approx 6.39$ sec

When $t = 6.39$, the velocity is approximately

$$\lim_{t \to a} \frac{s(a) - s(t)}{a - t} = \lim_{t \to a} -4.9(a + 4)$$
$$= \lim_{t \to 6.39} -4.9(6.39 + 4) = -50.9 \text{ m/sec.}$$

57. $\displaystyle\lim_{x \to 0} \frac{|x|}{x} = 1$ is false. $\displaystyle\lim_{x \to 0} \frac{|x|}{x}$ does not exist, since

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1 \quad \text{and} \quad \lim_{x \to 0^+} \frac{|x|}{x} = 1.$$

58. $\displaystyle\lim_{x \to 0} x^3 = 0$ is true.

59. True; see Theorem 1.7.

60. False; let $f(x) = \dfrac{x - 1}{x^2 - 1}$. Then,

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{1}{x + 1} = \frac{1}{2}$$

but $f(1)$ is not defined.

61. True

62. $\lim_{x \to 2} f(x) = 3$ is false since

$\lim_{x \to 2^-} f(x) = 3$ and $\lim_{x \to 2^+} f(x) = 0$.

63. $\lim_{x \to 3} f(x) = 1$ is true since

$\lim_{x \to 3^-} (x - 2) = 1$ and $\lim_{x \to 3^+} (-x^2 + 8x - 14) = 1$.

64. $f(x) = \dfrac{x^2 - 4}{|x - 2|} = (x + 2)\left[\dfrac{x - 2}{|x - 2|}\right]$

$\lim_{x \to 2^-} f(x) = -4$

$\lim_{x \to 2^+} f(x) = 4$

$\lim_{x \to 2} f(x)$ does not exist.

65. $f(x) = \sqrt{(x - 1)x}$

Domain: $(-\infty, 0] \cup [1, \infty)$

$\lim_{x \to 0^-} f(x) = 0$

$\lim_{x \to 1^+} f(x) = 0$