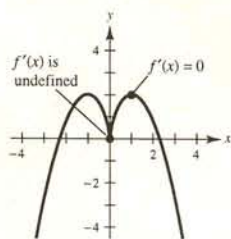


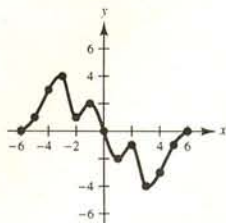
Review Exercises for Chapter 3

1. A number c in the domain of f is a critical number if $f'(c) = 0$ or f' is undefined at c .



2. (a) $f(4) = -f(-4) = -3$

(c)



At least six critical numbers on $(-6, 6)$.

3. $g(x) = 2x + 5 \cos x, [0, 2\pi]$

$$g'(x) = 2 - 5 \sin x$$

$$= 0 \text{ when } \sin x = \frac{2}{5}.$$

Critical numbers: $x \approx 0.41, x \approx 2.73$

Left endpoint: $(0, 5)$

Critical number: $(0.41, 5.41)$

Critical number: $(2.73, 0.88)$ Minimum

Right endpoint: $(2\pi, 17.57)$ Maximum

4. $f(x) = \frac{x}{\sqrt{x^2 + 1}}, [0, 2]$

$$f'(x) = x \left[-\frac{1}{2}(x^2 + 1)^{-3/2}(2x) \right] + (x^2 + 1)^{-1/2}$$

$$= \frac{1}{(x^2 + 1)^{3/2}}$$

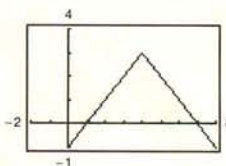
No critical numbers

Left endpoint: $(0, 0)$ Minimum

Right endpoint: $(2, 2/\sqrt{5})$ Maximum

5. $f(x) = 3 - |x - 4|$

(a)



$$f(1) = f(7) = 0$$

(b) f is not differentiable at $x = 4$.

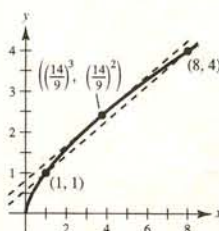
7. $f(x) = x^{2/3}, 1 \leq x \leq 8$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$\frac{f(b) - f(a)}{b - a} = \frac{4 - 1}{8 - 1} = \frac{3}{7}$$

$$f'(c) = \frac{2}{3}c^{-1/3} = \frac{3}{7}$$

$$c = \left(\frac{14}{9}\right)^3 = \frac{2744}{729} \approx 3.764$$



6. No; the function is discontinuous at $x = 0$ which is in the interval $[-2, 1]$.

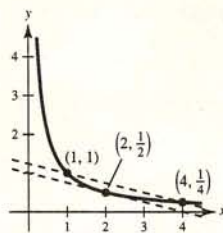
8. $f(x) = \frac{1}{x}, 1 \leq x \leq 4$

$$f'(x) = -\frac{1}{x^2}$$

$$\frac{f(b) - f(a)}{b - a} = \frac{(1/4) - 1}{4 - 1} = \frac{-3/4}{3} = -\frac{1}{4}$$

$$f'(c) = \frac{-1}{c^2} = -\frac{1}{4}$$

$$c = 2$$



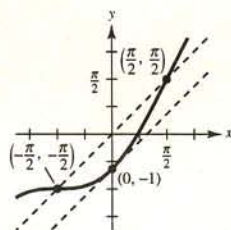
9. $f(x) = x - \cos x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

$$f'(x) = 1 + \sin x$$

$$\frac{f(b) - f(a)}{b - a} = \frac{(\pi/2) - (-\pi/2)}{(\pi/2) - (-\pi/2)} = 1$$

$$f'(c) = 1 + \sin c = 1$$

$$c = 0$$



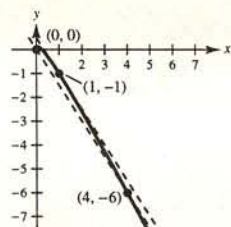
10. $f(x) = \sqrt{x} - 2x, 0 \leq x \leq 4$

$$f'(x) = \frac{1}{2\sqrt{x}} - 2$$

$$\frac{f(b) - f(a)}{b - a} = \frac{-6 - 0}{4 - 0} = -\frac{3}{2}$$

$$f'(c) = \frac{1}{2\sqrt{c}} - 2 = -\frac{3}{2}$$

$$c = 1$$



11. $f(x) = Ax^2 + Bx + C$

$$f'(x) = 2Ax + B$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{A(x_2^2 - x_1^2) + B(x_2 - x_1)}{x_2 - x_1}$$

$$= A(x_1 + x_2) + B$$

$$f'(c) = 2Ac + B = A(x_1 + x_2) + B$$

$$2Ac = A(x_1 + x_2)$$

$$c = \frac{x_1 + x_2}{2} = \text{Midpoint of } [x_1, x_2]$$

12. $f(x) = 2x^2 - 3x + 1$

$$f'(x) = 4x - 3$$

$$\frac{f(b) - f(a)}{b - a} = \frac{21 - 1}{4 - 0} = 5$$

$$f'(c) = 4c - 3 = 5$$

$$c = 2 = \text{Midpoint of } [0, 4]$$

13. $f(x) = (x - 1)^2(x - 3)$

$$f'(x) = (x - 1)^2(1) + (x - 3)(2)(x - 1)$$

$$= (x - 1)(3x - 7)$$

$$\text{Critical numbers: } x = 1 \text{ and } x = \frac{7}{3}$$

Interval	$-\infty < x < 1$	$1 < x < \frac{7}{3}$	$\frac{7}{3} < x < \infty$
Sign of $f'(x)$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

14. $g(x) = (x + 1)^3$

$$g'(x) = 3(x + 1)^2$$

$$\text{Critical number: } x = -1$$

Interval	$-\infty < x < -1$	$-1 < x < \infty$
Sign of $g'(x)$	$g'(x) > 0$	$g'(x) > 0$
Conclusion	Increasing	Increasing

15. $h(x) = \sqrt{x}(x-3) = x^{3/2} - 3x^{1/2}$

Domain: $[0, \infty)$

$$h'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$$

$$= \frac{3}{2}x^{-1/2}(x-1) = \frac{3(x-1)}{2\sqrt{x}}$$

Critical numbers: $x = 0, x = 1$

Interval	$0 < x < 1$	$1 < x < \infty$
Sign of $h'(x)$	$h'(x) < 0$	$h'(x) > 0$
Conclusion	Decreasing	Increasing

16. $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$

$$f'(x) = \cos x - \sin x$$

Critical numbers: $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

Interval	$0 < x < \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{5\pi}{4}$	$\frac{5\pi}{4} < x < 2\pi$
Sign of $f'(x)$	$f'(x) > 0$	$f'(x) < 0$	$f'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

17. $h(t) = \frac{1}{4}t^4 - 8t$

$$h'(t) = t^3 - 8 = 0 \text{ when } t = 2.$$

Relative minimum: $(2, -12)$

Test Interval	$-\infty < t < 2$	$2 < t < \infty$
Sign of $h'(t)$	$h'(t) < 0$	$h'(t) > 0$
Conclusion	Decreasing	Increasing

18. $g(x) = \frac{3}{2} \sin\left(\frac{\pi x}{2} - 1\right), [0, 4]$

$$g'(x) = \frac{3}{2} \left(\frac{\pi}{2}\right) \cos\left(\frac{\pi x}{2} - 1\right)$$

$$= 0 \text{ when } x = 1 + \frac{2}{\pi}, 3 + \frac{2}{\pi}$$

Relative maximum: $\left(1 + \frac{2}{\pi}, \frac{3}{2}\right)$

Relative minimum: $\left(3 + \frac{2}{\pi}, -\frac{3}{2}\right)$

Test Interval	$0 < x < 1 + \frac{2}{\pi}$	$1 + \frac{2}{\pi} < x < 3 + \frac{2}{\pi}$	$3 + \frac{2}{\pi} < x < 4$
Sign of $g'(x)$	$g'(x) > 0$	$g'(x) < 0$	$g'(x) > 0$
Conclusion	Increasing	Decreasing	Increasing

19. $f(x) = x + \cos x, 0 \leq x \leq 2\pi$

$$f'(x) = 1 - \sin x$$

$$f''(x) = -\cos x = 0 \text{ when } x = \frac{\pi}{2}, \frac{3\pi}{2}.$$

Points of inflection: $\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{3\pi}{2}, \frac{3\pi}{2}\right)$

Test Interval	$0 < x < \frac{\pi}{2}$	$\frac{\pi}{2} < x < \frac{3\pi}{2}$	$\frac{3\pi}{2} < x < 2\pi$
Sign of $f''(x)$	$f''(x) < 0$	$f''(x) > 0$	$f''(x) < 0$
Conclusion	Concave downward	Concave upward	Concave downward

20. $f(x) = (x+2)^2(x-4) = x^3 - 12x - 16$

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x = 0 \text{ when } x = 0.$$

Point of inflection: $(0, -16)$

Test Interval	$-\infty < x < 0$	$0 < x < \infty$
Sign of $f''(x)$	$f''(x) < 0$	$f''(x) > 0$
Conclusion	Concave downward	Concave upward

33. $f(x) = 4x - x^2 = x(4 - x)$

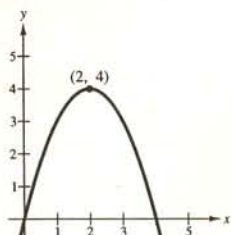
Domain: $(-\infty, \infty)$; Range: $(-\infty, 4)$

$$f'(x) = 4 - 2x = 0 \text{ when } x = 2.$$

$$f''(x) = -2$$

Therefore, $(2, 4)$ is a relative maximum.

Intercepts: $(0, 0)$, $(4, 0)$



34. $f(x) = 4x^3 - x^4 = x^3(4 - x)$

Domain: $(-\infty, \infty)$; Range: $(-\infty, 27)$

$$f'(x) = 12x^2 - 4x^3 = 4x^2(3 - x) = 0 \text{ when } x = 0, 3.$$

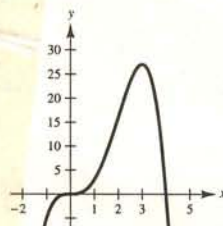
$$f''(x) = 24x - 12x^2 = 12x(2 - x) = 0 \text{ when } x = 0, 2.$$

$$f''(3) < 0$$

Therefore, $(3, 27)$ is a relative maximum.

Points of inflection: $(0, 0)$, $(2, 16)$

Intercepts: $(0, 0)$, $(4, 0)$



35. $f(x) = x\sqrt{16 - x^2}$

Domain: $[-4, 4]$; Range: $[-8, 8]$

$$f'(x) = \frac{16 - 2x^2}{\sqrt{16 - x^2}} = 0 \text{ when } x = \pm 2\sqrt{2} \text{ and undefined when } x = \pm 4.$$

$$f''(x) = \frac{2x(x^2 - 24)}{(16 - x^2)^{3/2}}$$

$$f''(-2\sqrt{2}) > 0$$

Therefore, $(-2\sqrt{2}, -8)$ is a relative minimum.

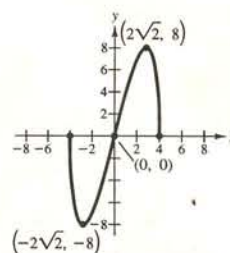
$$f''(2\sqrt{2}) < 0$$

Therefore, $(2\sqrt{2}, 8)$ is a relative maximum.

Point of inflection: $(0, 0)$

Intercepts: $(-4, 0)$, $(0, 0)$, $(4, 0)$

Symmetry with respect to origin



36. $f(x) = (x^2 - 4)^2$

Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

$$f'(x) = 4x(x^2 - 4) = 0 \text{ when } x = 0, \pm 2.$$

$$f''(x) = 4(3x^2 - 4) = 0 \text{ when } x = \pm \frac{2\sqrt{3}}{3}.$$

$$f''(0) < 0$$

Therefore, $(0, 16)$ is a relative maximum.

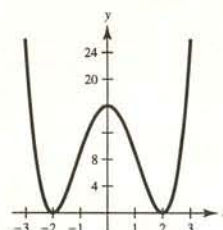
$$f''(\pm 2) > 0$$

Therefore, $(\pm 2, 0)$ are relative minima.

Points of inflection: $(\pm 2\sqrt{3}/3, 64/9)$

Intercepts: $(-2, 0)$, $(0, 16)$, $(2, 0)$

Symmetry with respect to y-axis



37. $f(x) = (x-1)^3(x-3)^2$

Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

$$f'(x) = (x-1)^2(x-3)(5x-11) = 0 \text{ when } x = 1, \frac{11}{5}, 3.$$

$$f''(x) = 4(x-1)(5x^2 - 22x + 23) = 0 \text{ when } x = 1, \frac{11 \pm \sqrt{6}}{5}.$$

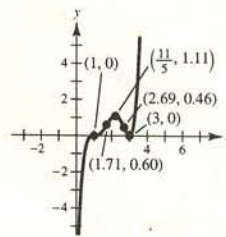
$$f''(3) > 0$$

Therefore, $(3, 0)$ is a relative minimum.

$$f''\left(\frac{11}{5}\right) < 0$$

Therefore, $\left(\frac{11}{5}, \frac{3456}{3125}\right)$ is a relative maximum.

$$\text{Points of inflection: } (1, 0), \left(\frac{11 - \sqrt{6}}{5}, 0.60\right), \left(\frac{11 + \sqrt{6}}{5}, 0.46\right)$$

Intercepts: $(0, -9), (1, 0), (3, 0)$ 

38. $f(x) = (x-3)(x+2)^3$

Domain: $(-\infty, \infty)$; Range: $\left[-\frac{16,875}{256}, \infty\right)$

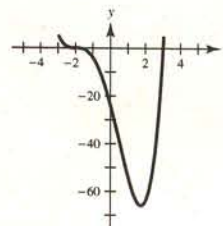
$$f'(x) = (x-3)(3)(x+2)^2 + (x+2)^3 \\ = (4x-7)(x+2)^2 = 0 \text{ when } x = -2, \frac{7}{4}.$$

$$f''(x) = (4x-7)(2)(x+2) + (x+2)^2(4) \\ = 6(2x-1)(x+2) = 0 \text{ when } x = -2, \frac{1}{2}.$$

$$f''\left(\frac{7}{4}\right) > 0$$

Therefore, $\left(\frac{7}{4}, -\frac{16,875}{256}\right)$ is a relative minimum.

$$\text{Points of inflection: } (-2, 0), \left(\frac{1}{2}, -\frac{625}{16}\right)$$

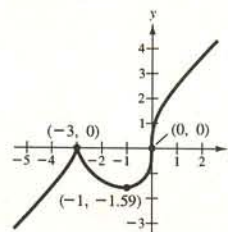
Intercepts: $(-2, 0), (0, -24), (3, 0)$ 

39. $f(x) = x^{1/3}(x+3)^{2/3}$

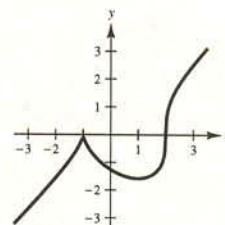
Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

$$f'(x) = \frac{x+1}{(x+3)^{1/3}x^{2/3}} = 0 \text{ when } x = -1 \text{ and undefined when } x = -3, 0.$$

$$f''(x) = \frac{-2}{x^{5/3}(x+3)^{4/3}} \text{ is undefined when } x = 0, -3.$$

By the First Derivative Test $(-3, 0)$ is a relative maximum and $(-1, -\sqrt[3]{4})$ is a relative minimum. $(0, 0)$ is a point of inflection.Intercepts: $(-3, 0), (0, 0)$ 

40. $f(x) = (x-2)^{1/3}(x+1)^{2/3}$

Graph of Exercise 39 translated 2 units to the right (x replaces by $x-2$). $(-1, 0)$ is a relative maximum. $(1, -\sqrt[3]{4})$ is a relative minimum. $(2, 0)$ is a point of inflection.Intercepts: $(-1, 0), (2, 0)$ 

41. $f(x) = \frac{x+1}{x-1}$

Domain: $(-\infty, 1), (1, \infty)$; Range: $(-\infty, 1), (1, \infty)$

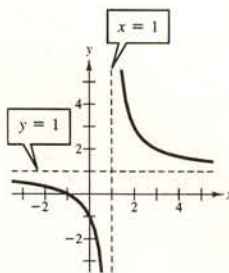
$$f'(x) = \frac{-2}{(x-1)^2} < 0 \text{ if } x \neq 1.$$

$$f''(x) = \frac{4}{(x-1)^3}$$

Horizontal asymptote: $y = 1$

Vertical asymptote: $x = 1$

Intercepts: $(-1, 0), (0, -1)$



42. $f(x) = \frac{2x}{1+x^2}$

Domain: $(-\infty, \infty)$; Range: $[-1, 1]$

$$f'(x) = \frac{2(1-x)(1+x)}{(1+x^2)^2} = 0 \text{ when } x = \pm 1.$$

$$f''(x) = \frac{-2x(3-x^2)}{(1+x^2)^3} = 0 \text{ when } x = 0, \pm\sqrt{3}.$$

$$f''(1) < 0$$

Therefore, $(1, 1)$ is a relative maximum.

$$f''(-1) > 0$$

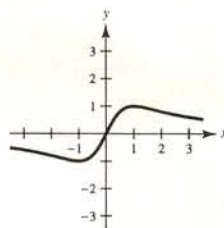
Therefore, $(-1, -1)$ is a relative minimum.

Points of inflection: $(-\sqrt{3}, -\sqrt{3}/2), (0, 0), (\sqrt{3}, \sqrt{3}/2)$

Intercept: $(0, 0)$

Symmetric with respect to the origin

Horizontal asymptote: $y = 0$



43. $f(x) = \frac{4}{1+x^2}$

Domain: $(-\infty, \infty)$; Range: $(0, 4]$

$$f'(x) = \frac{-8x}{(1+x^2)^2} = 0 \text{ when } x = 0.$$

$$f''(x) = \frac{-8(1-3x^2)}{(1+x^2)^3} = 0 \text{ when } x = \pm\frac{\sqrt{3}}{3}.$$

$$f''(0) < 0$$

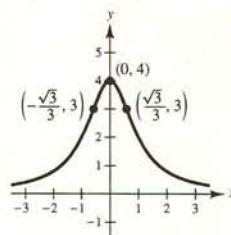
Therefore, $(0, 4)$ is a relative maximum.

Points of inflection: $(\pm\sqrt{3}/3, 3)$

Intercept: $(0, 4)$

Symmetric to the y-axis

Horizontal asymptote: $y = 0$



44. $f(x) = \frac{x^2}{1+x^4}$

Domain: $(-\infty, \infty)$; Range: $\left[0, \frac{1}{2}\right]$

$$f'(x) = \frac{(1+x^4)(2x) - x^2(4x^3)}{(1+x^4)^2} = \frac{2x(1-x)(1+x)(1+x^2)}{(1+x^4)^2} = 0 \text{ when } x = 0, \pm 1.$$

$$f''(x) = \frac{(1+x^4)^2(2-10x^4) - (2x-2x^5)(2)(1+x^4)(4x^3)}{(1+x^4)^4} = \frac{2(1-12x^4+3x^8)}{(1+x^4)^3} = 0 \text{ when } x = \pm \sqrt[4]{\frac{6 \pm \sqrt{33}}{3}}.$$

$$f''(\pm 1) < 0$$

Therefore, $\left(\pm 1, \frac{1}{2}\right)$ are relative maxima.

$$f''(0) > 0$$

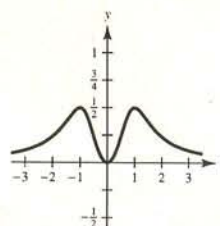
Therefore, $(0, 0)$ is a relative minimum.

$$\text{Points of inflection: } \left(\pm \sqrt[4]{\frac{6 - \sqrt{33}}{3}}, 0.29\right), \left(\pm \sqrt[4]{\frac{6 + \sqrt{33}}{3}}, 0.40\right)$$

Intercept: $(0, 0)$

Symmetric to the y-axis

Horizontal asymptote: $y = 0$



45. $f(x) = x^3 + x + \frac{4}{x}$

Domain: $(-\infty, 0), (0, \infty)$; Range: $(-\infty, -6], [6, \infty)$

$$f'(x) = 3x^2 + 1 - \frac{4}{x^2} = \frac{3x^4 + x^2 - 4}{x^2} = 0 \text{ when } x = \pm 1.$$

$$f''(x) = 6x + \frac{8}{x^3} = \frac{6x^4 + 8}{x^3} \neq 0$$

$$f''(-1) < 0$$

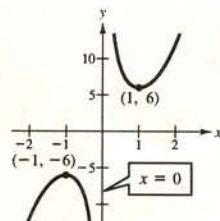
Therefore, $(-1, -6)$ is a relative maximum.

$$f''(1) > 0$$

Therefore, $(1, 6)$ is a relative minimum.

Vertical asymptote: $x = 0$

Symmetric with respect to origin



46. $f(x) = x^2 + \frac{1}{x} = \frac{x^3 + 1}{x}$

Domain: $(-\infty, 0), (0, \infty)$; Range: $(-\infty, \infty)$

$$f'(x) = 2x - \frac{1}{x^2} = \frac{2x^3 - 1}{x^2} = 0 \text{ when } x = \frac{1}{\sqrt[3]{2}}.$$

$$f''(x) = 2 + \frac{2}{x^3} = \frac{2(x^3 + 1)}{x^3} = 0 \text{ when } x = -1.$$

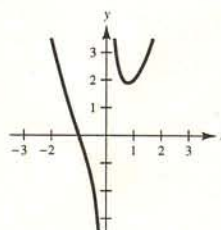
$$f''\left(\frac{1}{\sqrt[3]{2}}\right) > 0$$

Therefore, $\left(\frac{1}{\sqrt[3]{2}}, \frac{3}{\sqrt[3]{4}}\right)$ is a relative minimum.

Point of inflection: $(-1, 0)$

Intercept: $(-1, 0)$

Vertical asymptote: $x = 0$



47. $f(x) = |x^2 - 9|$

Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

$$f'(x) = \frac{2x(x^2 - 9)}{|x^2 - 9|} = 0 \text{ when } x = 0 \text{ and is undefined when } x = \pm 3.$$

$$f''(x) = \frac{2(x^2 - 9)}{|x^2 - 9|} \text{ is undefined at } x = \pm 3.$$

$$f''(0) < 0$$

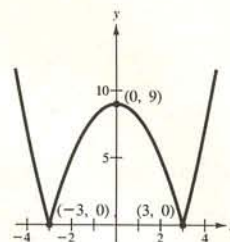
Therefore, $(0, 9)$ is a relative maximum.

Relative minima: $(\pm 3, 0)$

Points of inflection: $(\pm 3, 0)$

Intercepts: $(\pm 3, 0), (0, 9)$

Symmetric to the y-axis

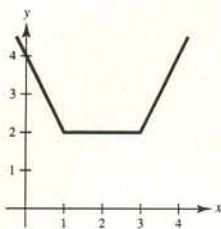


48. $f(x) = |x - 1| + |x - 3| = \begin{cases} -2x + 4, & x \leq 1 \\ 2, & 1 < x \leq 3 \\ 2x - 4, & x > 3 \end{cases}$

Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

Intercept: $(0, 4)$



49. $f(x) = x + \cos x$

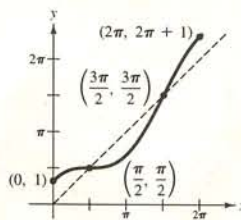
Domain: $[0, 2\pi]$; Range: $[1, 1 + 2\pi]$

$$f'(x) = 1 - \sin x \geq 0, f \text{ is increasing.}$$

$$f''(x) = -\cos x = 0 \text{ when } x = \frac{\pi}{2}, \frac{3\pi}{2}.$$

Points of inflection: $(\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{3\pi}{2})$

Intercept: $(0, 1)$



50. $f(x) = \frac{1}{\pi}(2 \sin \pi x - \sin 2\pi x)$

Domain: $[-1, 1]$; Range: $[\frac{-3\sqrt{3}}{2\pi}, \frac{3\sqrt{3}}{2\pi}]$

$$f'(x) = 2(\cos \pi x - \cos 2\pi x) = -2(2 \cos \pi x + 1)(\cos \pi x - 1) = 0$$

$$\text{Critical Numbers: } x = \pm \frac{2}{3}, 0.$$

$$f''(x) = 2\pi(-\sin \pi x + 2 \sin 2\pi x) = 2\pi \sin \pi x(-1 + 4 \cos \pi x) = 0 \text{ when } x = 0, \pm 1, \pm 0.420.$$

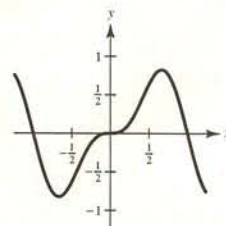
By the First Derivative Test: $(-\frac{2}{3}, \frac{-3\sqrt{3}}{2\pi})$ is a relative minimum.

$(\frac{2}{3}, \frac{3\sqrt{3}}{2\pi})$ is a relative maximum.

Points of inflection: $(-0.420, -0.462), (0.420, 0.462), (\pm 1, 0), (0, 0)$

Intercepts: $(-1, 0), (0, 0), (1, 0)$

Symmetric with respect to the origin



51. Let
- $t = 0$
- at noon.

$$L = d^2 = (100 - 12t)^2 + (-10t)^2 = 10,000 - 2400t + 244t^2$$

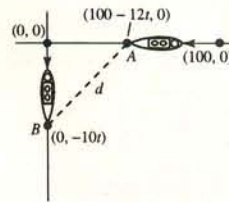
$$\frac{dL}{dt} = -2400 + 488t = 0 \text{ when } t = \frac{300}{61} \approx 4.92 \text{ hr.}$$

Ship A at (40.98, 0); Ship B at (0, -49.18)

$$d^2 = 10,000 - 2400t + 244t^2$$

$$\approx 4098.36 \text{ when } t \approx 4.92 \approx 4:55 \text{ P.M.}$$

$$d \approx 64 \text{ km}$$

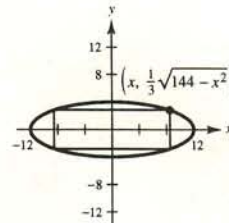


52. Ellipse:
- $\frac{x^2}{144} + \frac{y^2}{16} = 1$
- ,
- $y = \frac{1}{3}\sqrt{144 - x^2}$

$$A = (2x)\left(\frac{2}{3}\sqrt{144 - x^2}\right) = \frac{4}{3}x\sqrt{144 - x^2}$$

$$\frac{dA}{dx} = \frac{4}{3}\left[\frac{-x^2}{\sqrt{144 - x^2}} + \sqrt{144 - x^2}\right]$$

$$= \frac{4}{3}\left[\frac{144 - 2x^2}{\sqrt{144 - x^2}}\right] = 0 \text{ when } x = \sqrt{72} = 6\sqrt{2}.$$

The dimensions of the rectangle are $2x = 12\sqrt{2}$ by $y = \frac{2}{3}\sqrt{144 - 72} = 4\sqrt{2}$.

53. We have points (0, y), (x, 0), and (1, 8). Thus,

$$m = \frac{y - 8}{0 - 1} = \frac{0 - 8}{x - 1} \text{ or } y = \frac{8x}{x - 1}.$$

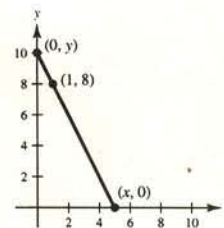
$$\text{Let } f(x) = L^2 = x^2 + \left(\frac{8x}{x - 1}\right)^2.$$

$$f'(x) = 2x + 128\left(\frac{x}{x - 1}\right)\left[\frac{(x - 1) - x}{(x - 1)^2}\right] = 0$$

$$x - \frac{64x}{(x - 1)^3} = 0$$

$$x[(x - 1)^3 - 64] = 0 \text{ when } x = 0, 5 \text{ (minimum).}$$

Vertices of triangle: (0, 0), (5, 0), (0, 10)



54. We have points (0, y), (x, 0), and (4, 5). Thus,

$$m = \frac{y - 5}{0 - 4} = \frac{5 - 0}{4 - x} \text{ or } y = \frac{5x}{x - 4}.$$

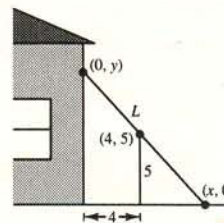
$$\text{Let } f(x) = L^2 = x^2 + \left(\frac{5x}{x - 4}\right)^2$$

$$f'(x) = 2x + 50\left(\frac{x}{x - 4}\right)\left[\frac{x - 4 - x}{(x - 4)^2}\right] = 0$$

$$x - \frac{100x}{(x - 4)^3} = 0$$

$$x[(x - 4)^3 - 100] = 0 \text{ when } x = 0 \text{ or } x = 4 + \sqrt[3]{100}.$$

$$L = \sqrt{x^2 + \frac{25x^2}{(x - 4)^2}} = \frac{x}{x - 4} \sqrt{(x - 4)^2 + 25} = \frac{\sqrt[3]{100} + 4}{\sqrt[3]{100}} \sqrt{100^{2/3} + 25} \approx 12.7 \text{ feet}$$



79. $x^2 + 4y^2 - 2x - 16y + 13 = 0$

(a) $(x^2 - 2x + 1) + 4(y^2 - 4y + 4) = -13 + 1 + 16$

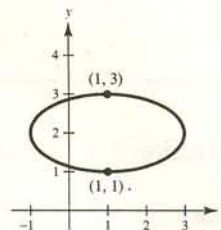
$$(x - 1)^2 + 4(y - 2)^2 = 4$$

$$\frac{(x - 1)^2}{4} + \frac{(y - 2)^2}{1} = 1$$

The graph is an ellipse:

Maximum: (1, 3)

Minimum: (1, 1)



(b) $x^2 + 4y^2 - 2x - 16y + 13 = 0$

$$2x + 8y \frac{dy}{dx} - 2 - 16 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(8y - 16) = 2 - 2x$$

$$\frac{dy}{dx} = \frac{2 - 2x}{8y - 16} = \frac{1 - x}{4y - 8}$$

The critical numbers are $x = 1$ and $y = 2$. These correspond to the points (1, 1), (1, 3), (2, -1), and (2, 3). Hence, the maximum is (1, 3) and the minimum is (1, 1).

80. $f(x) = x^n$, n is a positive integer.

(a) $f'(x) = nx^{n-1}$

The function has a relative minimum at (0, 0) when n is even.

(b) $f''(x) = n(n-1)x^{n-2}$

The function has a point of inflection at (0, 0) when n is odd and $n \geq 3$.

81. False

Let $f(x) = x^3$, $c = 0$.

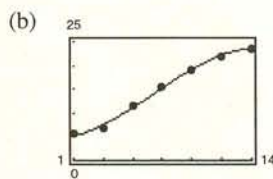
82. False

Let $f(x) = \frac{2x}{\sqrt{x^2 + 2}}$.

f has horizontal asymptotes at $y = 2$ and $y = -2$.

83. The first derivative is positive and the second derivative is negative. The graph is increasing and is concave down.

84. (a) $S = -0.1222t^3 + 1.3655t^2 - 0.9052t + 4.8429$



(c) $S'(t) = 0$ when $t = 3.7$. This is a maximum by the First Derivative Test.

(d) No, because the t^3 coefficient term is negative.