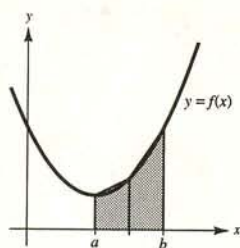
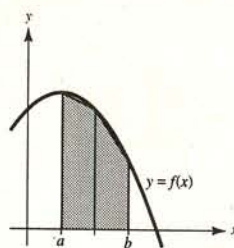


44. (a)



The Trapezoidal Rule overestimates the area if the graph of the integrand is concave up.

(b)



The Trapezoidal Rule underestimates the area if the graph of the integrand is concave down.

$$45. L(x) = \int_1^x \frac{1}{t} dt$$

$$(a) L(1) = \int_1^1 \frac{1}{t} dt = 0$$

$$(b) L'(x) = \frac{1}{x}$$

$$L'(1) = 1$$

(c) By trial and error, $x \approx 2.718$.

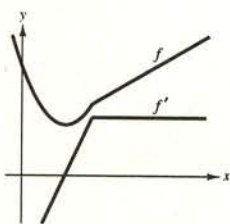
$$(d) L(x_1 x_2) = \int_1^{x_1 x_2} \frac{1}{t} dt = \int_1^{x_1} \frac{1}{t} dt + \int_{x_1}^{x_1 x_2} \frac{1}{t} dt$$

But, this second integral is equal to $\int_1^{x_2} (1/t) dt$, by substitution $u = t/x_1$, $du = dt/x_1$. Hence,

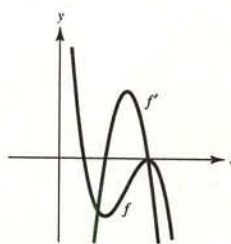
$$L(x_1 x_2) = L(x_1) + L(x_2).$$

Review Exercises for Chapter 4

1.



2.



$$3. \int (2x^2 + x - 1) dx = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$$

$$4. u = 3x$$

$$du = 3 dx$$

$$\int \frac{2}{\sqrt[3]{3x}} dx = \frac{2}{3} \int (3x)^{-1/3} (3) dx = (3x)^{2/3} + C$$

$$5. \int \frac{x^3 + 1}{x^2} dx = \int \left(x + \frac{1}{x^2} \right) dx = \frac{1}{2}x^2 - \frac{1}{x} + C$$

$$6. \int \frac{x^3 - 2x^2 + 1}{x^2} dx = \int (x - 2 + x^{-2}) dx = \frac{1}{2}x^2 - 2x - \frac{1}{x} + C$$

$$7. \int (4x - 3 \sin x) dx = 2x^2 + 3 \cos x + C$$

$$8. \int (5 \cos x - 2 \sec^2 x) dx = 5 \sin x - 2 \tan x + C$$

$$20. (a) \int_0^6 f(x) dx = \int_0^3 f(x) dx + \int_3^6 f(x) dx = 4 + (-1) = 3$$

$$(b) \int_6^3 f(x) dx = -\int_3^6 f(x) dx = -(-1) = 1$$

$$(c) \int_4^4 f(x) dx = 0$$

$$(d) \int_3^6 -10f(x) dx = -10 \int_3^6 f(x) dx = -10(-1) = 10$$

$$21. \int (x^2 + 1)^3 dx = \int (x^6 + 3x^4 + 3x^2 + 1) dx \\ = \frac{x^7}{7} + \frac{3}{5}x^5 + x^3 + x + C$$

$$22. \int \left(x + \frac{1}{x}\right)^2 dx = \int (x^2 + 2 + x^{-2}) dx \\ = \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

$$23. u = x^3 + 3, du = 3x^2 dx$$

$$\int \frac{x^2}{\sqrt{x^3 + 3}} dx = \int (x^3 + 3)^{-1/2} x^2 dx = \frac{1}{3} \int (x^3 + 3)^{-1/2} 3x^2 dx = \frac{2}{3}(x^3 + 3)^{1/2} + C$$

$$24. u = x^3 + 3, du = 3x^2 dx$$

$$\int x^2 \sqrt{x^3 + 3} dx = \frac{1}{3} \int (x^3 + 3)^{1/2} 3x^2 dx = \frac{2}{9}(x^3 + 3)^{3/2} + C$$

$$25. u = 1 - 3x^2, du = -6x dx$$

$$\int x(1 - 3x^2)^4 dx = -\frac{1}{6} \int (1 - 3x^2)^4 (-6x dx) = -\frac{1}{30}(1 - 3x^2)^5 + C = \frac{1}{30}(3x^2 - 1)^5 + C$$

$$26. u = x^2 + 6x - 5, du = (2x + 6) dx$$

$$\int \frac{x + 3}{(x^2 + 6x - 5)^2} dx = \frac{1}{2} \int \frac{2x + 6}{(x^2 + 6x - 5)^2} dx = \frac{-1}{2}(x^2 + 6x - 5)^{-1} + C = \frac{-1}{2(x^2 + 6x - 5)} + C$$

$$27. \int \sin^3 x \cos x dx = \frac{1}{4} \sin^4 x + C$$

$$28. \int x \sin 3x^2 dx = \frac{1}{6} \int (\sin 3x^2)(6x) dx = -\frac{1}{6} \cos 3x^2 + C$$

$$29. \int \frac{\sin \theta}{\sqrt{1 - \cos \theta}} d\theta = \int (1 - \cos \theta)^{-1/2} \sin \theta d\theta = 2(1 - \cos \theta)^{1/2} + C = 2\sqrt{1 - \cos \theta} + C$$

$$30. \int \frac{\cos x}{\sqrt{\sin x}} dx = \int (\sin x)^{-1/2} \cos x dx = 2(\sin x)^{1/2} + C = 2\sqrt{\sin x} + C$$

$$31. \int \tan^n x \sec^2 x dx = \frac{\tan^{n+1} x}{n+1} + C, n \neq -1$$

$$32. \int \sec 2x \tan 2x dx = \frac{1}{2} \int (\sec 2x \tan 2x)(2) dx = \frac{1}{2} \sec 2x + C$$

$$33. \int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x dx = \frac{1}{\pi} \int (1 + \sec \pi x)^2 (\pi \sec \pi x \tan \pi x) dx = \frac{1}{3\pi} (1 + \sec \pi x)^3 + C$$

$$34. \int \cot^4 \alpha \csc^2 \alpha \, d\alpha = -\int (\cot \alpha)^4 (-\csc^2 \alpha) \, d\alpha \\ = -\frac{1}{5} \cot^5 \alpha + C$$

$$35. \int_0^4 (2+x) \, dx = \left[2x + \frac{x^2}{2} \right]_0^4 = 8 + \frac{16}{2} = 16$$

$$36. \int_{-1}^1 (t^2 + 2) \, dt = \left[\frac{t^3}{3} + 2t \right]_{-1}^1 = \frac{14}{3}$$

$$37. \int_{-1}^1 (4t^3 - 2t) \, dt = \left[t^4 - t^2 \right]_{-1}^1 = 0$$

$$38. \int_3^6 \frac{x}{3\sqrt{x^2-8}} \, dx = \frac{1}{6} \int_3^6 (x^2-8)^{-1/2} (2x) \, dx = \left[\frac{1}{3} (x^2-8)^{1/2} \right]_3^6 = \frac{1}{3} (2\sqrt{7} - 1)$$

$$39. \int_0^3 \frac{1}{\sqrt{1+x}} \, dx = \int_0^3 (1+x)^{-1/2} \, dx = \left[2(1+x)^{1/2} \right]_0^3 = 4 - 2 = 2$$

$$40. \int_0^1 x^2(x^3+1)^3 \, dx = \frac{1}{3} \int_0^1 (x^3+1)^3 (3x^2) \, dx = \frac{1}{12} \left[(x^3+1)^4 \right]_0^1 = \frac{1}{12} (16-1) = \frac{5}{4}$$

$$41. \int_4^9 x\sqrt{x} \, dx = \int_4^9 x^{3/2} \, dx = \left[\frac{2}{5} x^{5/2} \right]_4^9 = \frac{2}{5} [(\sqrt{9})^5 - (\sqrt{4})^5] = \frac{2}{5} (243 - 32) = \frac{422}{5}$$

$$42. \int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3} \right) \, dx = \int_1^2 (x^{-2} - x^{-3}) \, dx = \left[-\frac{1}{x} + \frac{1}{2x^2} \right]_1^2 = \left(-\frac{1}{2} + \frac{1}{8} \right) - \left(-1 + \frac{1}{2} \right) = \frac{1}{8}$$

$$43. u = 1 - y, y = 1 - u, dy = -du$$

When $y = 0$, $u = 1$. When $y = 1$, $u = 0$.

$$2\pi \int_0^1 (y+1)\sqrt{1-y} \, dy = 2\pi \int_1^0 -[(1-u)+1]\sqrt{u} \, du \\ = 2\pi \int_1^0 (u^{3/2} - 2u^{1/2}) \, du = 2\pi \left[\frac{2}{5} u^{5/2} - \frac{4}{3} u^{3/2} \right]_1^0 = \frac{28\pi}{15}$$

$$44. u = x + 1, x = u - 1, dx = du$$

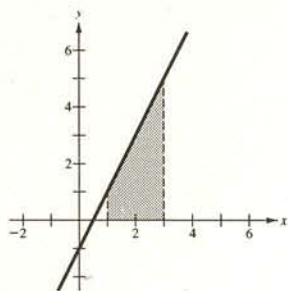
When $x = -1$, $u = 0$. When $x = 0$, $u = 1$.

$$2\pi \int_{-1}^0 x^2 \sqrt{x+1} \, dx = 2\pi \int_0^1 (u-1)^2 \sqrt{u} \, du \\ = 2\pi \int_0^1 (u^{5/2} - 2u^{3/2} + u^{1/2}) \, du = 2\pi \left[\frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]_0^1 = \frac{32\pi}{105}$$

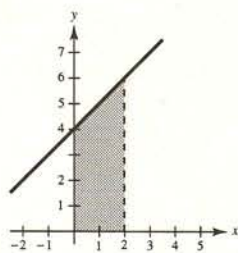
$$45. \int_0^\pi \cos\left(\frac{x}{2}\right) \, dx = 2 \int_0^\pi \cos\left(\frac{x}{2}\right) \frac{1}{2} \, dx = \left[2 \sin\left(\frac{x}{2}\right) \right]_0^\pi = 2$$

$$46. \int_{-\pi/4}^{\pi/4} \sin 2x \, dx = 0 \text{ since } \sin 2x \text{ is an odd function.}$$

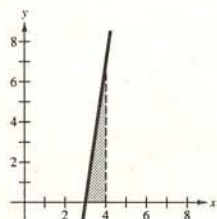
$$47. \int_1^3 (2x - 1) dx = \left[x^2 - x \right]_1^3 = 6$$



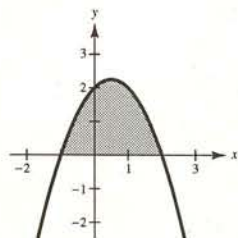
$$48. \int_0^2 (x + 4) dx = \left[\frac{x^2}{2} + 4x \right]_0^2 = 10$$



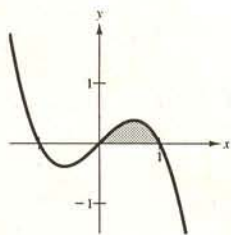
$$\begin{aligned} 49. \int_3^4 (x^2 - 9) dx &= \left[\frac{x^3}{3} - 9x \right]_3^4 \\ &= \left(\frac{64}{3} - 36 \right) - (9 - 27) \\ &= \frac{64}{3} - \frac{54}{3} = \frac{10}{3} \end{aligned}$$



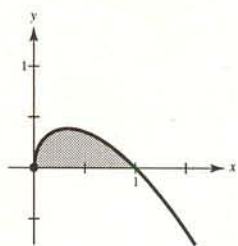
$$\begin{aligned} 50. \int_{-1}^2 (-x^2 + x + 2) dx &= \left[-\frac{x^3}{3} + \frac{x^2}{2} + 2x \right]_{-1}^2 \\ &= \left(-\frac{8}{3} + 2 + 4 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right) \\ &= \frac{10}{3} + \frac{7}{6} = \frac{9}{2} \end{aligned}$$



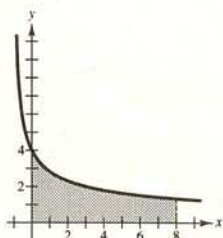
$$\begin{aligned} 51. \int_0^1 (x - x^3) dx &= \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 \\ &= \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \end{aligned}$$



$$\begin{aligned} 52. \int_0^1 \sqrt{x}(1-x) dx &= \int_0^1 (x^{1/2} - x^{3/2}) dx \\ &= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} \right]_0^1 \\ &= \frac{2}{3} - \frac{2}{5} = \frac{4}{15} \end{aligned}$$

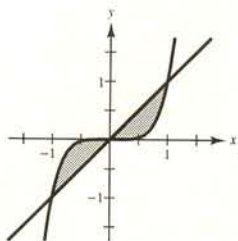


$$\begin{aligned} 53. \int_0^8 \frac{4}{\sqrt{x+1}} dx &= 4 \int_0^8 (x+1)^{-1/2} dx \\ &= \left[8\sqrt{x+1} \right]_0^8 \\ &= 8(3 - 1) = 16 \end{aligned}$$

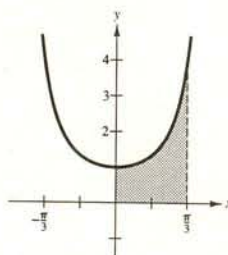


$$54. \int_{-1}^0 (x^5 - x) dx + \int_0^1 (x - x^5) dx = 2 \int_0^1 (x - x^5) dx$$

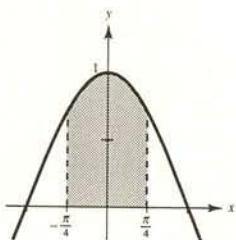
$$= 2 \left[\frac{x^2}{2} - \frac{x^6}{6} \right]_0^1 = \frac{2}{3}$$



$$55. \int_0^{\pi/3} \sec^2 x dx = [\tan x]_0^{\pi/3} = \sqrt{3}$$



$$56. \int_{-\pi/4}^{\pi/4} \cos x dx = 2 \int_0^{\pi/4} \cos x dx = [2 \sin x]_0^{\pi/4} = \sqrt{2}$$



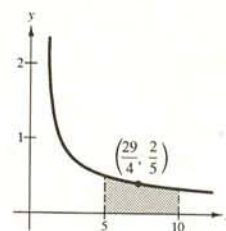
$$57. \frac{1}{10-5} \int_5^{10} \frac{1}{\sqrt{x-1}} dx = \left[\frac{2}{5} \sqrt{x-1} \right]_5^{10} = \frac{2}{5}$$

$$\frac{1}{\sqrt{x-1}} = \frac{2}{5}$$

$$\sqrt{x-1} = \frac{5}{2}$$

$$x-1 = \frac{25}{4}$$

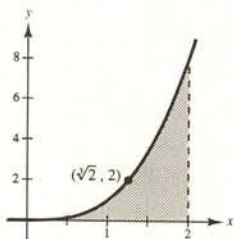
$$x = \frac{29}{4}$$



$$58. \frac{1}{2-0} \int_0^2 x^3 dx = \left[\frac{x^4}{8} \right]_0^2 = 2$$

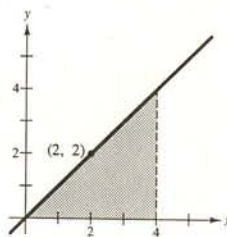
$$x^3 = 2$$

$$x = \sqrt[3]{2}$$



$$59. \frac{1}{4-0} \int_0^4 x dx = \left[\frac{x^2}{8} \right]_0^4 = 2$$

$$x = 2$$



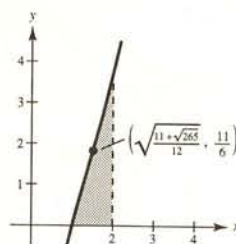
$$60. \frac{1}{2-1} \int_1^2 \left(x^2 - \frac{1}{x^2} \right) dx = \left[\frac{x^3}{3} + \frac{1}{x} \right]_1^2 = \left(\frac{8}{3} + \frac{1}{2} \right) - \left(\frac{1}{3} + 1 \right) = \frac{11}{6}$$

$$x^2 - \frac{1}{x^2} = \frac{11}{6}$$

$$6x^4 - 6 = 11x^2$$

$$6x^4 - 11x^2 - 6 = 0 \text{ when } x^2 = \frac{11 \pm \sqrt{265}}{12}$$

$$\text{In the interval } [1, 2]: x = \sqrt{\frac{11 + \sqrt{265}}{12}} \approx 1.508.$$



19. $u = 7x - 2, du = 7dx$

$$\int \frac{1}{7x-2} dx = \frac{1}{7} \int \frac{1}{7x-2} (7) dx = \frac{1}{7} \ln|7x-2| + C$$

21. $\int \frac{\sin x}{1 + \cos x} dx = -\int \frac{-\sin x}{1 + \cos x} dx$
 $= -\ln|1 + \cos x| + C$

23. $\int_1^4 \frac{x+1}{x} dx = \int_1^4 \left(1 + \frac{1}{x}\right) dx = \left[x + \ln|x|\right]_1^4 = 3 + \ln 4$

25. $\int_0^{\pi/3} \sec \theta d\theta = \left[\ln|\sec \theta + \tan \theta|\right]_0^{\pi/3} = \ln(2 + \sqrt{3})$

27. (a) $f(x) = \frac{1}{2}x - 3$
 $y = \frac{1}{2}x - 3$
 $2(y + 3) = x$
 $2(x + 3) = y$
 $f^{-1}(x) = 2x + 6$

28. (a) $f(x) = 5x - 7$
 $y = 5x - 7$
 $\frac{y+7}{5} = x$
 $\frac{x+7}{5} = y$
 $f^{-1}(x) = \frac{x+7}{5}$

29. (a) $f(x) = \sqrt{x+1}$
 $y = \sqrt{x+1}$
 $y^2 - 1 = x$
 $x^2 - 1 = y$
 $f^{-1}(x) = x^2 - 1, x \geq 0$

20. $u = x^2 - 1, du = 2x dx$

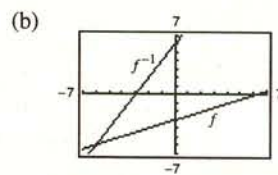
$$\int \frac{x}{x^2-1} dx = \frac{1}{2} \int \frac{2x}{x^2-1} dx = \frac{1}{2} \ln|x^2-1| + C$$

22. $u = \ln x, du = \frac{1}{x} dx$

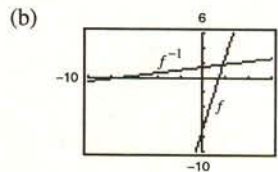
$$\int \frac{\ln \sqrt{x}}{x} dx = \frac{1}{2} \int (\ln x) \left(\frac{1}{x}\right) dx = \frac{1}{4} (\ln x)^2 + C$$

24. $\int_1^e \frac{\ln x}{x} dx = \int_1^e (\ln x)^1 \left(\frac{1}{x}\right) dx = \left[\frac{1}{2} (\ln x)^2\right]_1^e = \frac{1}{2}$

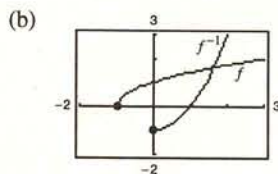
26. $\int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx = \left[\ln\left|\cos\left(\frac{\pi}{4} - x\right)\right|\right]_0^{\pi/4}$
 $= 0 - \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \ln 2$



(c) $f^{-1}(f(x)) = f^{-1}\left(\frac{1}{2}x - 3\right) = 2\left(\frac{1}{2}x - 3\right) + 6 = x$
 $f(f^{-1}(x)) = f(2x + 6) = \frac{1}{2}(2x + 6) - 3 = x$



(c) $f^{-1}(f(x)) = f^{-1}(5x - 7) = \frac{(5x - 7) + 7}{5} = x$
 $f(f^{-1}(x)) = f\left(\frac{x+7}{5}\right) = 5\left(\frac{x+7}{5}\right) - 7 = x$



(c) $f^{-1}(f(x)) = f^{-1}(\sqrt{x+1}) = \sqrt{(x^2-1)^2} - 1 = x$
 $f(f^{-1}(x)) = f(x^2 - 1) = \sqrt{(x^2 - 1) + 1}$
 $= \sqrt{x^2} = x \text{ for } x \geq 0.$

62. $10,000 = Pe^{(0.07)(15)}$

$$P = \frac{10,000}{e^{1.05}} \approx \$3499.38$$

63. $2P = Pe^{10r}$

$$2 = e^{10r}$$

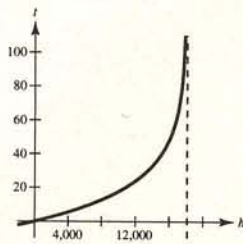
$$\ln 2 = 10r$$

$$r = \frac{\ln 2}{10} \approx 6.93\%$$

64. $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

(a) Domain: $0 \leq h < 18,000$

(b)



Vertical asymptote: $h = 18,000$

(c) $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

$$10^{t/50} = \frac{18,000}{18,000 - h}$$

$$18,000 - h = 18,000(10^{-t/50})$$

$$h = 18,000(1 - 10^{-t/50})$$

As $h \rightarrow 18,000$, $t \rightarrow \infty$.

(d) $t = 50 \log_{10} 18,000 - 50 \log_{10}(18,000 - h)$

$$\frac{dt}{dh} = \frac{50}{(\ln 10)(18,000 - h)}$$

$$\frac{d^2t}{dh^2} = \frac{50}{(\ln 10)(18,000 - h)^2}$$

No critical numbers

As t increases, the rate of change of the altitude is increasing.

65. Let $u = -3x^2$, $du = -6x dx$.

$$\int xe^{-3x^2} dx = -\frac{1}{6} \int e^{-3x^2} (-6x) dx = -\frac{1}{6} e^{-3x^2} + C$$

66. Let $u = \frac{1}{x}$, $du = -\frac{1}{x^2} dx$.

$$\int \frac{e^{1/x}}{x^2} dx = - \int e^{1/x} \left(-\frac{1}{x^2} \right) dx = -e^{1/x} + C$$

67. $\int \frac{e^{4x} - e^{2x} + 1}{e^x} dx = \int (e^{3x} - e^x + e^{-x}) dx$

$$= \frac{1}{3} e^{3x} - e^x - e^{-x} + C$$

$$= \frac{e^{4x} - 3e^{2x} - 3}{3e^x} + C$$

68. Let $u = e^{2x} + e^{-2x}$, $du = (2e^{2x} - e^{-2x}) dx$.

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx = \frac{1}{2} \int \frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}} dx$$

$$= \frac{1}{2} \ln(e^{2x} + e^{-2x}) + C$$

69. Let $u = e^x - 1$, $du = e^x dx$.

$$\int \frac{e^x}{e^x - 1} dx = \ln|e^x - 1| + C$$

70. Let $u = x^3 + 1$, $du = 3x^2 dx$.

$$\int x^2 e^{x^3+1} dx = \frac{1}{3} \int e^{x^3+1} (3x^2) dx = \frac{1}{3} e^{x^3+1} + C$$

71. Let $u = e^{2x}$, $du = 2e^{2x} dx$.

$$\int \frac{1}{e^{2x} + e^{-2x}} dx = \int \frac{e^{2x}}{1 + e^{4x}} dx = \frac{1}{2} \int \frac{1}{1 + (e^{2x})^2} (2e^{2x}) dx = \frac{1}{2} \arctan(e^{2x}) + C$$

72. Let $u = 5x$, $du = 5 dx$.

$$\int \frac{1}{3 + 25x^2} dx = \frac{1}{5} \int \frac{1}{(\sqrt{3})^2 + (5x)^2} (5) dx = \frac{1}{5\sqrt{3}} \arctan \frac{5x}{\sqrt{3}} + C$$

73. Let $u = x^2$, $du = 2x dx$.

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \int \frac{1}{\sqrt{1-(x^2)^2}} (2x) dx = \frac{1}{2} \arcsin x^2 + C$$

75. Let $u = 16 + x^2$, $du = 2x dx$.

$$\int \frac{x}{16+x^2} dx = \frac{1}{2} \int \frac{1}{16+x^2} (2x) dx = \frac{1}{2} \ln(16+x^2) + C$$

76. $\int \frac{4-x}{\sqrt{4-x^2}} dx = 4 \int \frac{1}{\sqrt{4-x^2}} dx + \frac{1}{2} \int (4-x^2)^{-1/2} (-2x) dx = 4 \arcsin \frac{x}{2} + \sqrt{4-x^2} + C$

77. Let $u = \arctan\left(\frac{x}{2}\right)$, $du = \frac{2}{4+x^2} dx$.

$$\int \frac{\arctan(x/2)}{4+x^2} dx = \frac{1}{2} \int \left(\arctan \frac{x}{2} \right) \left(\frac{2}{4+x^2} \right) dx = \frac{1}{4} \left(\arctan \frac{x}{2} \right)^2 + C$$

78. Let $u = \arcsin x$, $du = \frac{1}{\sqrt{1-x^2}} dx$.

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx = \frac{1}{2} (\arcsin x)^2 + C$$

79. Let $u = x^2$, $du = 2x dx$.

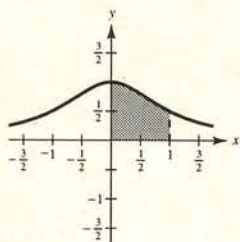
$$\int \frac{x}{\sqrt{x^4-1}} dx = \frac{1}{2} \int \frac{1}{\sqrt{(x^2)^2-1}} (2x) dx = \frac{1}{2} \ln(x^2 + \sqrt{x^4-1}) + C$$

80. Let $u = x^3$, $du = 3x^2 dx$.

$$\int x^2 (\operatorname{sech} x^3)^2 dx = \frac{1}{3} \int (\operatorname{sech} x^3)^2 (3x^2) dx = \frac{1}{3} \tanh x^3 + C$$

81. Area = $\int_0^4 x e^{-x^2} dx = \left[-\frac{1}{2} e^{-x^2} \right]_0^4 = -\frac{1}{2} (e^{-16} - 1) \approx 0.500$

82. Area = $\int_0^1 \frac{1}{x^2+1} dx = \left[\arctan x \right]_0^1 = \arctan 1 = \frac{\pi}{4}$



74. $\int \frac{1}{16+x^2} dx = \frac{1}{4} \arctan \frac{x}{4} + C$

83. $\frac{dy}{dx} = \frac{x^2+3}{x}$

$$\int dy = \int \left(x + \frac{3}{x} \right) dx$$

$$y = \frac{x^2}{2} + 3 \ln|x| + C$$

$$84. \frac{dy}{dx} = \frac{e^{-2x}}{1 + e^{-2x}}$$

$$\int dy = \int \frac{e^{-2x}}{1 + e^{-2x}} dx = -\frac{1}{2} \int \frac{-2e^{-2x}}{1 + e^{-2x}} dx$$

$$y = -\frac{1}{2} \ln(1 + e^{-2x}) + C$$

$$85. y' - 2xy = 0$$

$$\frac{dy}{dx} = 2xy$$

$$\int \frac{1}{y} dy = \int 2x dx$$

$$\ln|y| = x^2 + C_1$$

$$e^{x^2+C_1} = y$$

$$y = Ce^{x^2}$$

$$86. y' - e^y \sin x = 0$$

$$\frac{dy}{dx} = e^y \sin x$$

$$\int e^{-y} dy = \int \sin x dx$$

$$-e^{-y} = -\cos x + C_1$$

$$e^y = \frac{1}{\cos x + C} \quad (C = -C_1)$$

$$y = \ln \left| \frac{1}{\cos x + C} \right| = -\ln|\cos x + C|$$

$$87. \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \quad (\text{homogeneous differential equation})$$

$$(x^2 + y^2) dx - 2xy dy = 0$$

$$\text{Let } y = vx, dy = x dv + v dx.$$

$$(x^2 + v^2x^2) dx - 2x(vx)(x dv + v dx) = 0$$

$$(x^2 + v^2x^2 - 2x^2v^2) dx - 2x^3v dv = 0$$

$$(x^2 - x^2v^2) dx = 2x^3v dv$$

$$(1 - v^2) dx = 2x dv$$

$$\int \frac{dx}{x} = \int \frac{2v}{1 - v^2} dv$$

$$\ln|x| = -\ln|1 - v^2| + C_1 = -\ln|1 - v^2| + \ln C$$

$$x = \frac{C}{1 - v^2} = \frac{C}{1 - (y/x)^2} = \frac{Cx^2}{x^2 - y^2}$$

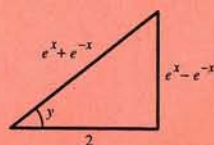
$$1 = \frac{Cx}{x^2 - y^2} \quad \text{or} \quad C_1 = \frac{x}{x^2 - y^2}$$

81. As k increases, the time required for the object to reach the ground increases.

82. Let $y = \arcsin(\tanh x)$. Then,

$$\sin y = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \text{ and}$$

$$\tan y = \frac{e^x - e^{-x}}{2} = \sinh x.$$



83. $y = \cosh x = \frac{e^x + e^{-x}}{2}$

$$y' = \frac{e^x - e^{-x}}{2} = \sinh x$$

Thus, $y = \arctan(\sinh x)$. Therefore,

$$\arctan(\sinh x) = \arcsin(\tanh x).$$

84. $y = \operatorname{sech}^{-1} x$

$$\operatorname{sech} y = x$$

$$-(\operatorname{sech} y)(\tanh y)y' = 1$$

$$y' = \frac{-1}{(\operatorname{sech} y)(\tanh y)} = \frac{-1}{(\operatorname{sech} y)\sqrt{1 - \operatorname{sech}^2 y}} = \frac{-1}{x\sqrt{1 - x^2}}$$

85. $y = \cosh^{-1} x$

$$\cosh y = x$$

$$(\sinh y)(y') = 1$$

$$y' = \frac{1}{\sinh y} = \frac{1}{\sqrt{\cosh^2 y - 1}} = \frac{1}{\sqrt{x^2 - 1}}$$

86. $y = \sinh^{-1} x$

$$\sinh y = x$$

$$(\cosh y)y' = 1$$

$$y' = \frac{1}{\cosh y} = \frac{1}{\sqrt{\sinh^2 y + 1}} = \frac{1}{\sqrt{x^2 + 1}}$$

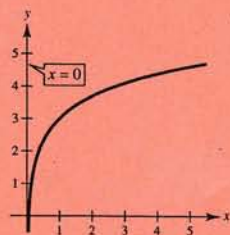
87. $y = \operatorname{sech} x = \frac{2}{e^x + e^{-x}}$

$$y' = -2(e^x + e^{-x})^{-2}(e^x - e^{-x}) = \left(\frac{-2}{e^x + e^{-x}}\right)\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right) = -\operatorname{sech} x \tanh x$$

Review Exercises for Chapter 5

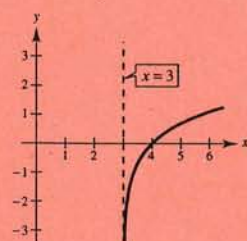
1. $f(x) = \ln x + 3$

Vertical shift 3 units upward
Vertical asymptote: $x = 0$



2. $f(x) = \ln(x - 3)$

Horizontal shift 3 units to the right
Vertical asymptote: $x = 3$



3. $\ln \sqrt[5]{\frac{4x^2 - 1}{4x^2 + 1}} = \frac{1}{5} \ln \frac{(2x - 1)(2x + 1)}{4x^2 + 1} = \frac{1}{5} [\ln(2x - 1) + \ln(2x + 1) - \ln(4x^2 + 1)]$

4. $\ln[(x^2 + 1)(x - 1)] = \ln(x^2 + 1) + \ln(x - 1)$

5. $\ln 3 + \frac{1}{3} \ln(4 - x^2) - \ln x = \ln 3 + \ln \sqrt[3]{4 - x^2} - \ln x = \ln \left(\frac{3\sqrt[3]{4 - x^2}}{x} \right)$

$$6. 3[\ln x - 2 \ln(x^2 + 1)] + 2 \ln 5 = 3 \ln x - 6 \ln(x^2 + 1) + \ln 5^2 = \ln x^3 - \ln(x^2 + 1)^6 + \ln 25 = \ln \left[\frac{25x^3}{(x^2 + 1)^6} \right]$$

7. False; the domain of $f(x) = \ln x$ is the set of all **positive** real numbers.

$$9. \ln \sqrt{x+1} = 2$$

$$\sqrt{x+1} = e^2$$

$$x+1 = e^4$$

$$x = e^4 - 1 \approx 53.598$$

$$11. g(x) = \ln \sqrt{x} = \frac{1}{2} \ln x$$

$$g'(x) = \frac{1}{2x}$$

$$13. f(x) = x\sqrt{\ln x}$$

$$f'(x) = \left(\frac{x}{2}\right)(\ln x)^{-1/2}\left(\frac{1}{x}\right) + \sqrt{\ln x}$$

$$= \frac{1}{2\sqrt{\ln x}} + \sqrt{\ln x} = \frac{1 + 2 \ln x}{2\sqrt{\ln x}}$$

$$15. y = \frac{1}{b^2} \left[\ln(a + bx) + \frac{a}{a + bx} \right]$$

$$\frac{dy}{dx} = \frac{1}{b^2} \left[\frac{b}{a + bx} - \frac{ab}{(a + bx)^2} \right] = \frac{x}{(a + bx)^2}$$

$$17. y = -\frac{1}{a} \ln \left(\frac{a + bx}{x} \right) = -\frac{1}{a} [\ln(a + bx) - \ln x]$$

$$\frac{dy}{dx} = -\frac{1}{a} \left(\frac{b}{a + bx} - \frac{1}{x} \right) = \frac{1}{x(a + bx)}$$

8. False

$$\ln x + \ln y = \ln(xy) \neq \ln(x + y)$$

$$10. \ln x + \ln(x - 3) = 0$$

$$\ln x(x - 3) = 0$$

$$x(x - 3) = e^0$$

$$x^2 - 3x - 1 = 0$$

$$x = \frac{3 \pm \sqrt{13}}{2}$$

$$x = \frac{3 + \sqrt{13}}{2} \text{ only since } \frac{3 - \sqrt{13}}{2} < 0.$$

$$12. h(x) = \ln \frac{x(x-1)}{x-2} = \ln x + \ln(x-1) - \ln(x-2)$$

$$h'(x) = \frac{1}{x} + \frac{1}{x-1} - \frac{1}{x-2} = \frac{x^2 - 4x + 2}{x^3 - 3x^2 + 2x}$$

$$14. f(x) = \ln[x(x^2 - 2)^{2/3}] = \ln x + \frac{2}{3} \ln(x^2 - 2)$$

$$f'(x) = \frac{1}{x} + \frac{2}{3} \left(\frac{2x}{x^2 - 2} \right) = \frac{7x^2 - 6}{3x^3 - 6x}$$

$$16. y = \frac{1}{b^2} [a + bx - a \ln(a + bx)]$$

$$\frac{dy}{dx} = \frac{1}{b^2} \left(b - \frac{ab}{a + bx} \right) = \frac{x}{a + bx}$$

$$18. y = -\frac{1}{ax} + \frac{b}{a^2} \ln \frac{a + bx}{x}$$

$$= -\frac{1}{ax} + \frac{b}{a^2} [\ln(a + bx) - \ln x]$$

$$\frac{dy}{dx} = -\frac{1}{a} \left(-\frac{1}{x^2} \right) + \frac{b}{a^2} \left[\frac{b}{a + bx} - \frac{1}{x} \right]$$

$$= \frac{1}{ax^2} + \frac{b}{a^2} \left[\frac{-a}{x(a + bx)} \right] = \frac{1}{ax^2} - \frac{b}{ax(a + bx)}$$

$$= \frac{(a + bx) - bx}{ax^2(a + bx)} = \frac{1}{x^2(a + bx)}$$

19. $u = 7x - 2, du = 7dx$

$$\int \frac{1}{7x-2} dx = \frac{1}{7} \int \frac{1}{7x-2} (7) dx = \frac{1}{7} \ln|7x-2| + C$$

$$\begin{aligned} 21. \int \frac{\sin x}{1 + \cos x} dx &= - \int \frac{-\sin x}{1 + \cos x} dx \\ &= -\ln|1 + \cos x| + C \end{aligned}$$

$$23. \int_1^4 \frac{x+1}{x} dx = \int_1^4 \left(1 + \frac{1}{x}\right) dx = \left[x + \ln|x|\right]_1^4 = 3 + \ln 4$$

$$25. \int_0^{\pi/3} \sec \theta d\theta = \left[\ln|\sec \theta + \tan \theta|\right]_0^{\pi/3} = \ln(2 + \sqrt{3})$$

$$\begin{aligned} 27. (a) \quad f(x) &= \frac{1}{2}x - 3 \\ y &= \frac{1}{2}x - 3 \\ 2(y + 3) &= x \\ 2(x + 3) &= y \\ f^{-1}(x) &= 2x + 6 \end{aligned}$$

$$\begin{aligned} 28. (a) \quad f(x) &= 5x - 7 \\ y &= 5x - 7 \\ \frac{y+7}{5} &= x \\ \frac{x+7}{5} &= y \\ f^{-1}(x) &= \frac{x+7}{5} \end{aligned}$$

$$\begin{aligned} 29. (a) \quad f(x) &= \sqrt{x+1} \\ y &= \sqrt{x+1} \\ y^2 - 1 &= x \\ x^2 - 1 &= y \\ f^{-1}(x) &= x^2 - 1, x \geq 0 \end{aligned}$$

20. $u = x^2 - 1, du = 2x dx$

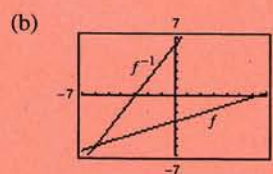
$$\int \frac{x}{x^2-1} dx = \frac{1}{2} \int \frac{2x}{x^2-1} dx = \frac{1}{2} \ln|x^2-1| + C$$

22. $u = \ln x, du = \frac{1}{x} dx$

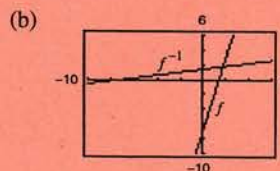
$$\int \frac{\ln \sqrt{x}}{x} dx = \frac{1}{2} \int (\ln x) \left(\frac{1}{x}\right) dx = \frac{1}{4} (\ln x)^2 + C$$

$$24. \int_1^e \frac{\ln x}{x} dx = \int_1^e (\ln x) \left(\frac{1}{x}\right) dx = \left[\frac{1}{2} (\ln x)^2\right]_1^e = \frac{1}{2}$$

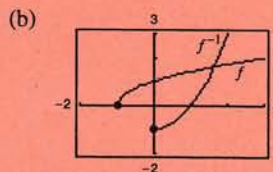
$$\begin{aligned} 26. \int_0^{\pi/4} \tan\left(\frac{\pi}{4} - x\right) dx &= \left[\ln\left|\cos\left(\frac{\pi}{4} - x\right)\right|\right]_0^{\pi/4} \\ &= 0 - \ln\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} \ln 2 \end{aligned}$$



$$\begin{aligned} (c) \quad f^{-1}(f(x)) &= f^{-1}\left(\frac{1}{2}x - 3\right) = 2\left(\frac{1}{2}x - 3\right) + 6 = x \\ f(f^{-1}(x)) &= f(2x + 6) = \frac{1}{2}(2x + 6) - 3 = x \end{aligned}$$

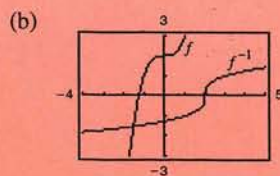


$$\begin{aligned} (c) \quad f^{-1}(f(x)) &= f^{-1}(5x - 7) = \frac{(5x - 7) + 7}{5} = x \\ f(f^{-1}(x)) &= f\left(\frac{x+7}{5}\right) = 5\left(\frac{x+7}{5}\right) - 7 = x \end{aligned}$$



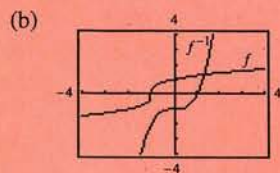
$$\begin{aligned} (c) \quad f^{-1}(f(x)) &= f^{-1}(\sqrt{x+1}) = \sqrt{(x+1)-1} = x \\ f(f^{-1}(x)) &= f(x^2 - 1) = \sqrt{(x^2 - 1) + 1} \\ &= \sqrt{x^2} = x \text{ for } x \geq 0. \end{aligned}$$

30. (a) $f(x) = x^3 + 2$
 $y = x^3 + 2$
 $\sqrt[3]{y-2} = x$
 $\sqrt[3]{x-2} = y$
 $f^{-1}(x) = \sqrt[3]{x-2}$



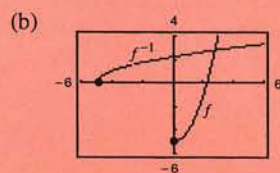
(c) $f^{-1}(f(x)) = f^{-1}(x^3 + 2) = \sqrt[3]{(x^3 + 2) - 2} = x$
 $f(f^{-1}(x)) = f(\sqrt[3]{x-2}) = (\sqrt[3]{x-2})^3 + 2 = x$

31. (a) $f(x) = \sqrt[3]{x+1}$
 $y = \sqrt[3]{x+1}$
 $y^3 - 1 = x$
 $x^3 - 1 = y$
 $f^{-1}(x) = x^3 - 1$



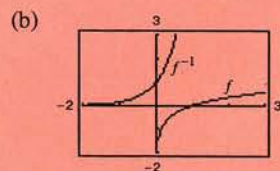
(c) $f^{-1}(f(x)) = f^{-1}(\sqrt[3]{x+1}) = (\sqrt[3]{x+1})^3 - 1 = x$
 $f(f^{-1}(x)) = f(x^3 - 1) = \sqrt[3]{(x^3 - 1) + 1} = x$

32. (a) $f(x) = x^2 - 5, x \geq 0$
 $y = x^2 - 5$
 $\sqrt{y+5} = x$
 $\sqrt{x+5} = y$
 $f^{-1}(x) = \sqrt{x+5}$



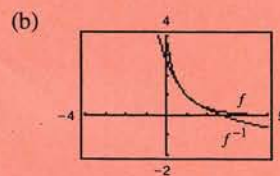
(c) $f^{-1}(f(x)) = f^{-1}(x^2 - 5) = \sqrt{(x^2 - 5) + 5} = x$ for $x \geq 0$.
 $f(f^{-1}(x)) = f(\sqrt{x+5}) = (\sqrt{x+5})^2 - 5 = x$

33. (a) $f(x) = \ln \sqrt{x}$
 $y = \ln \sqrt{x}$
 $e^y = \sqrt{x}$
 $e^{2y} = x$
 $e^{2x} = y$
 $f^{-1}(x) = e^{2x}$



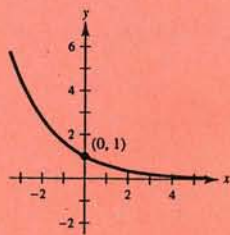
(c) $f^{-1}(f(x)) = f^{-1}(\ln \sqrt{x}) = e^{2 \ln \sqrt{x}} = e^{\ln x} = x$
 $f(f^{-1}(x)) = f(e^{2x}) = \ln \sqrt{e^{2x}} = \ln e^x = x$

34. (a) $f(x) = e^{1-x}$
 $y = e^{1-x}$
 $\ln y = 1 - x$
 $x = 1 - \ln y$
 $y = 1 - \ln x$
 $f^{-1}(x) = 1 - \ln x$

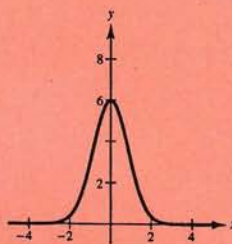


(c) $f^{-1}(f(x)) = f^{-1}(e^{1-x}) = 1 - \ln(e^{1-x})$
 $= 1 - (1 - x) = x$
 $f(f^{-1}(x)) = f(1 - \ln x) = e^{1 - (1 - \ln x)} = e^{\ln x} = x$

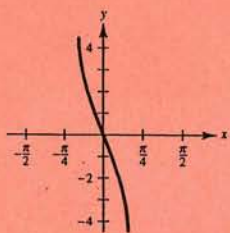
35. $y = e^{-x/2}$



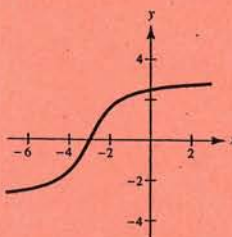
36. $g(x) = 6(2^{-x^2})$



37. $h(x) = -3 \arcsin(2x)$



38. $f(x) = 2 \arctan(x + 3)$



39. (a) Let $\theta = \arcsin \frac{1}{2}$

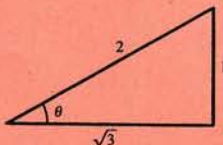
$$\sin \theta = \frac{1}{2}$$

$$\sin\left(\arcsin \frac{1}{2}\right) = \sin \theta = \frac{1}{2}$$

(b) Let $\theta = \arcsin \frac{1}{2}$

$$\sin \theta = \frac{1}{2}$$

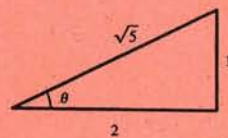
$$\cos\left(\arcsin \frac{1}{2}\right) = \cos \theta = \frac{\sqrt{3}}{2}$$



40. (a) Let $\theta = \operatorname{arccot} 2$

$$\cot \theta = 2$$

$$\tan(\operatorname{arccot} 2) = \tan \theta = \frac{1}{2}$$



(b) Let $\theta = \operatorname{arcsec} \sqrt{5}$

$$\sec \theta = \sqrt{5}$$

$$\cos(\operatorname{arcsec} \sqrt{5}) = \cos \theta = \frac{1}{\sqrt{5}}$$



41. $f(x) = \ln(e^{-x^2}) = -x^2$

$$f'(x) = -2x$$

42. $g(x) = \ln\left(\frac{e^x}{1+e^x}\right)$

$$= \ln e^x - \ln(1+e^x) = x - \ln(1+e^x)$$

$$g'(x) = 1 - \frac{e^x}{1+e^x} = \frac{1}{1+e^x}$$

43. $g(t) = t^2 e^t$

$$g'(x) = t^2 e^t + 2te^t = te^t(t+2)$$

44. $h(z) = e^{-z^2/2}$

$$h'(z) = -ze^{-z^2/2}$$

45. $y = \sqrt{e^{2x} + e^{-2x}}$

$$y' = \frac{1}{2}(e^{2x} + e^{-2x})^{-1/2}(2e^{2x} - 2e^{-2x}) = \frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$$

46. $y = x^{2x+1}$

$$\ln y = (2x+1) \ln x$$

$$\frac{y'}{y} = \frac{2x+1}{x} + 2 \ln x$$

$$y' = y\left(\frac{2x+1}{x} + 2 \ln x\right) = x^{2x+1}\left(\frac{2x+1}{x} + 2 \ln x\right)$$

$$47. f(x) = 3^{x-1}$$

$$f'(x) = 3^{x-1} \ln 3$$

$$49. g(x) = \frac{x^2}{e^x}$$

$$g'(x) = \frac{e^x(2x) - x^2 e^x}{e^{2x}} = \frac{x(2-x)}{e^x}$$

$$51. y = \tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$$

$$y' = \frac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{1-x^2} = (1-x^2)^{-3/2}$$

$$53. y = x \operatorname{arcsec} x$$

$$y' = \frac{x}{|x|\sqrt{x^2-1}} + \operatorname{arcsec} x$$

$$55. y = x(\arcsin x)^2 - 2x + 2\sqrt{1-x^2} \arcsin x$$

$$y' = \frac{2x \arcsin x}{\sqrt{1-x^2}} + (\arcsin x)^2 - 2 + \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}} - \frac{2x}{\sqrt{1-x^2}} \arcsin x = (\arcsin x)^2$$

$$56. y = \sqrt{x^2-4} - 2 \operatorname{arcsec} \frac{x}{2}, 2 < x < 4$$

$$y' = \frac{x}{\sqrt{x^2-4}} - \frac{1}{(|x|/2)\sqrt{(x/2)^2-1}} = \frac{x}{\sqrt{x^2-4}} - \frac{4}{|x|\sqrt{x^2-4}} = \frac{x^2-4}{|x|\sqrt{x^2-4}} = \frac{\sqrt{x^2-4}}{x}$$

$$57. y = 2x - \cosh \sqrt{x}$$

$$y' = 2 - \frac{1}{2\sqrt{x}}(\sinh \sqrt{x}) = 2 - \frac{\sinh \sqrt{x}}{2\sqrt{x}}$$

$$59. y(\ln x) + y^2 = 0$$

$$y\left(\frac{1}{x}\right) + (\ln x)\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$(2y + \ln x)\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-y}{x(2y + \ln x)}$$

$$61. (a) y = a^x \quad (b) y = a^x$$

$$y' = ax^{a-1} \quad y' = (\ln a)a^x$$

$$48. f(x) = 4^x e^x$$

$$f'(x) = 4^x e^x + (\ln 4)4^x e^x = 4^x e^x(1 + \ln 4)$$

$$50. f(\theta) = \frac{1}{2}e^{\sin 2\theta}$$

$$f'(\theta) = \cos 2\theta e^{\sin 2\theta}$$

$$52. y = \arctan(x^2 - 1)$$

$$y' = \frac{2x}{1 + (x^2 - 1)^2} = \frac{2x}{x^4 - 2x^2 + 2}$$

$$54. y = \frac{1}{2} \arctan e^{2x}$$

$$y' = \frac{1}{2} \left(\frac{1}{1 + e^{4x}} \right) (2e^{2x}) = \frac{e^{2x}}{1 + e^{4x}}$$

$$58. y = x \tanh^{-1} 2x$$

$$y' = x \left(\frac{2}{1-4x^2} \right) + \tanh^{-1} 2x = \frac{2x}{1-4x^2} + \tanh^{-1} 2x$$

$$60. \cos x^2 = xe^y$$

$$-2x \sin x^2 = xe^y \frac{dy}{dx} + e^y$$

$$\frac{dy}{dx} = -\frac{2x \sin x^2 + e^y}{xe^y}$$

$$(c) y = x^x \quad (d) y = a^a$$

$$\ln y = x \ln x \quad y' = 0$$

$$\frac{1}{y} y' = x \cdot \frac{1}{x} + (1) \ln x$$

$$y' = y(1 + \ln x)$$

$$y' = x^x(1 + \ln x)$$

$$62. 10,000 = Pe^{(0.07)(15)}$$

$$P = \frac{10,000}{e^{1.05}} \approx \$3499.38$$

$$63. 2P = Pe^{10r}$$

$$2 = e^{10r}$$

$$\ln 2 = 10r$$

$$r = \frac{\ln 2}{10} \approx 6.93\%$$

62. $10,000 = Pe^{(0.07)(15)}$

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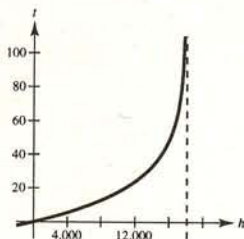
$$\ln 2 = 10r$$

$$r = \frac{\ln 2}{10} \approx 6.93\%$$

64. $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

(a) Domain: $0 \leq h < 18,000$

(b)



Vertical asymptote: $h = 18,000$

(c) $t = 50 \log_{10} \left(\frac{18,000}{18,000 - h} \right)$

$$10^{t/50} = \frac{18,000}{18,000 - h}$$

$$18,000 - h = 18,000(10^{-t/50})$$

$$h = 18,000(1 - 10^{-t/50})$$

As $h \rightarrow 18,000$, $t \rightarrow \infty$.

(d) $t = 50 \log_{10} 18,000 - 50 \log_{10}(18,000 - h)$

$$\frac{dt}{dh} = \frac{50}{(\ln 10)(18,000 - h)}$$

$$\frac{d^2t}{dh^2} = \frac{50}{(\ln 10)(18,000 - h)^2}$$

No critical numbers

As t increases, the rate of change of the altitude is increasing.

65. Let $u = -3x^2$, $du = -6x dx$.

$$\int xe^{-3x^2} dx = -\frac{1}{6} \int e^{-3x^2} (-6x) dx = -\frac{1}{6} e^{-3x^2} + C$$

66. Let $u = \frac{1}{x}$, $du = -\frac{1}{x^2} dx$.

$$\int \frac{e^{1/x}}{x^2} dx = -\int e^{1/x} \left(-\frac{1}{x^2} \right) dx = -e^{1/x} + C$$

67. $\int \frac{e^{4x} - e^{2x} + 1}{e^x} dx = \int (e^{3x} - e^x + e^{-x}) dx$

$$= \frac{1}{3} e^{3x} - e^x - e^{-x} + C$$

$$= \frac{e^{4x} - 3e^{2x} - 3}{3e^x} + C$$

68. Let $u = e^{2x} + e^{-2x}$, $du = (2e^{2x} - e^{-2x}) dx$.

$$\int \frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}} dx = \frac{1}{2} \int \frac{2e^{2x} - 2e^{-2x}}{e^{2x} + e^{-2x}} dx$$

$$= \frac{1}{2} \ln(e^{2x} + e^{-2x}) + C$$

69. Let $u = e^x - 1$, $du = e^x dx$.

$$\int \frac{e^x}{e^x - 1} dx = \ln|e^x - 1| + C$$

70. Let $u = x^3 + 1$, $du = 3x^2 dx$.

$$\int x^2 e^{x^3+1} dx = \frac{1}{3} \int e^{x^3+1} (3x^2) dx = \frac{1}{3} e^{x^3+1} + C$$

71. Let $u = e^{2x}$, $du = 2e^{2x} dx$.

$$\int \frac{1}{e^{2x} + e^{-2x}} dx = \int \frac{e^{2x}}{1 + e^{4x}} dx = \frac{1}{2} \int \frac{1}{1 + (e^{2x})^2} (2e^{2x}) dx = \frac{1}{2} \arctan(e^{2x}) + C$$

72. Let $u = 5x$, $du = 5 dx$.

$$\int \frac{1}{3 + 25x^2} dx = \frac{1}{5} \int \frac{1}{(\sqrt{3})^2 + (5x)^2} (5) dx = \frac{1}{5\sqrt{3}} \arctan \frac{5x}{\sqrt{3}} + C$$

73. Let $u = x^2$, $du = 2x dx$.

$$\int \frac{x}{\sqrt{1-x^4}} dx = \frac{1}{2} \int \frac{1}{\sqrt{1-(x^2)^2}} (2x) dx = \frac{1}{2} \arcsin x^2 + C$$

74. $\int \frac{1}{16+x^2} dx = \frac{1}{4} \arctan \frac{x}{4} + C$

75. Let $u = 16 + x^2$, $du = 2x dx$.

$$\int \frac{x}{16+x^2} dx = \frac{1}{2} \int \frac{1}{16+x^2} (2x) dx = \frac{1}{2} \ln(16+x^2) + C$$

76. $\int \frac{4-x}{\sqrt{4-x^2}} dx = 4 \int \frac{1}{\sqrt{4-x^2}} dx + \frac{1}{2} \int (4-x^2)^{-1/2} (-2x) dx = 4 \arcsin \frac{x}{2} + \sqrt{4-x^2} + C$

77. Let $u = \arctan\left(\frac{x}{2}\right)$, $du = \frac{2}{4+x^2} dx$.

$$\int \frac{\arctan(x/2)}{4+x^2} dx = \frac{1}{2} \int \left(\arctan \frac{x}{2} \right) \left(\frac{2}{4+x^2} \right) dx = \frac{1}{4} \left(\arctan \frac{x}{2} \right)^2 + C$$

78. Let $u = \arcsin x$, $du = \frac{1}{\sqrt{1-x^2}} dx$.

$$\int \frac{\arcsin x}{\sqrt{1-x^2}} dx = \frac{1}{2} (\arcsin x)^2 + C$$

79. Let $u = x^2$, $du = 2x dx$.

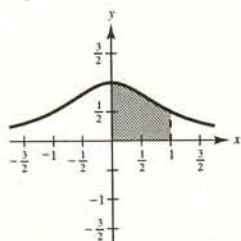
$$\int \frac{x}{\sqrt{x^4-1}} dx = \frac{1}{2} \int \frac{1}{\sqrt{(x^2)^2-1}} (2x) dx = \frac{1}{2} \ln(x^2 + \sqrt{x^4-1}) + C$$

80. Let $u = x^3$, $du = 3x^2 dx$.

$$\int x^2 (\operatorname{sech} x^3)^2 dx = \frac{1}{3} \int (\operatorname{sech} x^3)^2 (3x^2) dx = \frac{1}{3} \tanh x^3 + C$$

81. Area = $\int_0^4 x e^{-x^2} dx = \left[-\frac{1}{2} e^{-x^2} \right]_0^4 = -\frac{1}{2} (e^{-16} - 1) \approx 0.500$

82. Area = $\int_0^1 \frac{1}{x^2+1} dx = \left[\arctan x \right]_0^1 = \arctan 1 = \frac{\pi}{4}$



83. $\frac{dy}{dx} = \frac{x^2+3}{x}$

$$\int dy = \int \left(x + \frac{3}{x} \right) dx$$

$$y = \frac{x^2}{2} + 3 \ln|x| + C$$

$$84. \frac{dy}{dx} = \frac{e^{-2x}}{1 + e^{-2x}}$$

$$\int dy = \int \frac{e^{-2x}}{1 + e^{-2x}} dx = -\frac{1}{2} \int \frac{-2e^{-2x}}{1 + e^{-2x}} dx$$

$$y = -\frac{1}{2} \ln(1 + e^{-2x}) + C$$

$$85. y' - 2xy = 0$$

$$\frac{dy}{dx} = 2xy$$

$$\int \frac{1}{y} dy = \int 2x dx$$

$$\ln|y| = x^2 + C_1$$

$$e^{x^2+C_1} = y$$

$$y = Ce^{x^2}$$

$$86. y' - e^y \sin x = 0$$

$$\frac{dy}{dx} = e^y \sin x$$

$$\int e^{-y} dy = \int \sin x dx$$

$$-e^{-y} = -\cos x + C_1$$

$$e^y = \frac{1}{\cos x + C} \quad (C = -C_1)$$

$$y = \ln \left| \frac{1}{\cos x + C} \right| = -\ln|\cos x + C|$$

$$87. \frac{dy}{dx} = \frac{x^2 + y^2}{2xy} \quad (\text{homogeneous differential equation})$$

$$(x^2 + y^2) dx - 2xy dy = 0$$

$$\text{Let } y = vx, dy = x dv + v dx.$$

$$(x^2 + v^2x^2) dx - 2x(vx)(x dv + v dx) = 0$$

$$(x^2 + v^2x^2 - 2x^2v^2) dx - 2x^3v dv = 0$$

$$(x^2 - x^2v^2) dx = 2x^3v dv$$

$$(1 - v^2) dx = 2x dv$$

$$\int \frac{dx}{x} = \int \frac{2v}{1 - v^2} dv$$

$$\ln|x| = -\ln|1 - v^2| + C_1 = -\ln|1 - v^2| + \ln C$$

$$x = \frac{C}{1 - v^2} = \frac{C}{1 - (y/x)^2} = \frac{Cx^2}{x^2 - y^2}$$

$$1 = \frac{Cx}{x^2 - y^2} \quad \text{or} \quad C_1 = \frac{x}{x^2 - y^2}$$