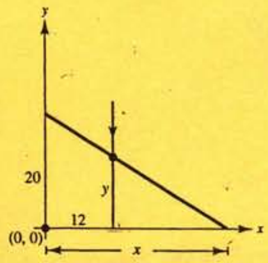
52. $y(t) = -4.9t^2 + 20$

$$\frac{dy}{dt} = -9.8t$$

$$y(1) = -4.9 + 20 = 15.1$$

$$y'(1) = -9.8$$



By similar triangles, $\dfrac{20}{x} = \dfrac{y}{x - 12}$

$$20x - 240 = xy.$$

When $y = 15.1$, $20x - 240 = x(15.1)$

$$(20 - 15.1)x = 240$$

$$x = \frac{240}{4.9}.$$

$$20x - 240 = xy$$

$$20\frac{dx}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{x}{20 - y}\frac{dy}{dt}$$

At $t = 1$, $\dfrac{dx}{dt} = \dfrac{240/4.9}{20 - 15.1}(-9.8) \approx -97.96$ m/sec.

Review Exercises for Chapter 2

1. $f(x) = x^2 - 2x + 3$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{[(x + \Delta x)^2 - 2(x + \Delta x) + 3] - [x^2 - 2x + 3]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x^2 + 2x(\Delta x) + (\Delta x)^2 - 2x - 2(\Delta x) + 3) - (x^2 - 2x + 3)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x(\Delta x) + (\Delta x)^2 - 2(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 2) = 2x - 2$$

2. $f(x) = \dfrac{x + 1}{x - 1}$

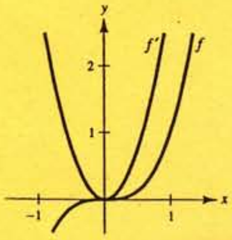
$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{x + \Delta x + 1}{x + \Delta x - 1} - \dfrac{x + 1}{x - 1}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x + 1)(x - 1) - (x + \Delta x - 1)(x + 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

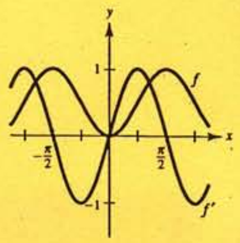
$$= \lim_{\Delta x \to 0} \frac{(x^2 + x\Delta x + x - x - \Delta x - 1) - (x^2 + x\Delta x - x + x + \Delta x - 1)}{\Delta x(x + \Delta x - 1)(x - 1)}$$

$$= \lim_{\Delta x \to 0} \frac{-2\Delta x}{\Delta x(x + \Delta x - 1)(x - 1)} = \lim_{\Delta x \to 0} \frac{-2}{(x + \Delta x - 1)(x - 1)} = \frac{-2}{(x - 1)^2}$$

3. (a)



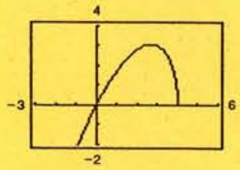
(b)



$f' > 0$ where the slopes of tangent lines to the graph of f are positive.

4. $f(x) = x\sqrt{4-x}$

(a)



(b) $f'(x) = x\left(\dfrac{1}{2}\right)(4-x)^{-1/2}(-1) + (4-x)^{1/2}$

$= \dfrac{1}{2}(4-x)^{-1/2}[-x + 2(4-x)]$

$= \dfrac{8-3x}{2\sqrt{4-x}}$

When $x = 0$, $f'(0) = 2$.

Tangent line: $y = 2x$

(c) $x = 0$

Δx	$f(x+\Delta x)$	$f(x)$	$\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$
2	2.8284	0	1.4142
1	1.7321	0	1.732
0.5	0.9354	0	1.8708
0.1	0.1975	0	1.9748

(d) $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ represents the slope of the line through $(x, f(x))$ and $(x + \Delta x, f(x + \Delta x))$. As $\Delta x \to 0$, this quantity approaches $f'(x)$. That is, the numbers in the last column approach $f'(0) = 2$.

5. f is differentiable for all $x \neq 1$.

6. f is differentiable for all $x \neq -3$.

7. $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 3x(x-2)$

8. $f(x) = x^{1/2} - x^{-1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}x^{-3/2} = \dfrac{x+1}{2x^{3/2}}$

9. $f(x) = 2x - x^{-2}$

$f'(x) = 2 + 2x^{-3} = 2\left(1 + \dfrac{1}{x^3}\right)$

$= \dfrac{2(x^3+1)}{x^3}$

10. $f(x) = \dfrac{x+1}{x-1}$

$f'(x) = \dfrac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$

$= \dfrac{-2}{(x-1)^2}$

11. $g(t) = \dfrac{2}{3}t^{-2}$

$g'(x) = \dfrac{-4}{3}t^{-3} = \dfrac{-4}{3t^3}$

12. $h(x) = \dfrac{2}{9}x^{-2}$

$h'(x) = \dfrac{-4}{9}x^{-3} = \dfrac{-4}{9x^3}$

13. $f(x) = (1 - x^3)^{1/2}$

$f'(x) = \dfrac{1}{2}(1-x^3)^{-1/2}(-3x^2)$

$= -\dfrac{3x^2}{2\sqrt{1-x^3}}$

14. $f(x) = (x^2 - 1)^{1/3}$

$f'(x) = \dfrac{1}{3}(x^2-1)^{-2/3}(2x)$

$= \dfrac{2x}{3(x^2-1)^{2/3}}$

15. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

$$f'(x) = (3x^2 + 7)(2x - 2) + (x^2 - 2x + 3)(6x)$$
$$= 2(6x^3 - 9x^2 + 16x - 7)$$

16. $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$

$$f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)\left(\tfrac{5}{2}\right)(s^2 - 1)^{3/2}(2s)$$
$$= s(s^2 - 1)^{3/2}[3s(s^2 - 1) + 5(s^3 + 5)]$$
$$= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$$

17. $f(x) = \left(x^2 + \dfrac{1}{x}\right)^5$

$$f'(x) = 5\left(x^2 + \frac{1}{x}\right)^4\left(2x - \frac{1}{x^2}\right)$$

18. $h(\theta) = \dfrac{\theta}{(1 - \theta)^3}$

$$h'(\theta) = \frac{(1 - \theta)^3 - \theta[3(1 - \theta)^2(-1)]}{(1 - \theta)^6}$$
$$= \frac{(1 - \theta)^2(1 - \theta + 3\theta)}{(1 - \theta)^6} = \frac{2\theta + 1}{(1 - \theta)^4}$$

19. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

$$f'(x) = \frac{(x^2 - 1)(2x + 1) - (x^2 + x - 1)(2x)}{(x^2 - 1)^2}$$
$$= \frac{-(x^2 + 1)}{(x^2 - 1)^2}$$

20. $f(x) = \dfrac{6x - 5}{x^2 + 1}$

$$f'(x) = \frac{(x^2 + 1)(6) - (6x - 5)(2x)}{(x^2 + 1)^2}$$
$$= \frac{2(3 + 5x - 3x^2)}{(x^2 + 1)^2}$$

21. $f(x) = (4 - 3x^2)^{-1}$

$$f'(x) = -(4 - 3x^2)^{-2}(-6x) = \frac{6x}{(4 - 3x^2)^2}$$

22. $f(x) = 9(3x^2 - 2x)^{-1}$

$$f'(x) = -9(3x^2 - 2x)^{-2}(6x - 2) = \frac{18(1 - 3x)}{(3x^2 - 2x)^2}$$

23. $y = 3\cos(3x + 1)$

$$y' = -9\sin(3x + 1)$$

24. $y = 1 - \cos 2x + 2\cos^2 x$

$$y' = 2\sin 2x - 4\cos x \sin x$$
$$= 2[2\sin x\cos x] - 4\sin x\cos x$$
$$= 0$$

25. $y = \dfrac{1}{2}\csc 2x$

$$y' = \frac{1}{2}(-\csc 2x \cot 2x)(2)$$
$$= -\csc 2x \cot 2x$$

26. $y = \csc 3x + \cot 3x$

$$y' = -3\csc 3x \cot 3x - 3\csc^2 3x$$
$$= -3\csc 3x(\cot 3x + \csc 3x)$$

27. $y = \dfrac{x}{2} - \dfrac{\sin 2x}{4}$

$$y' = \frac{1}{2} - \frac{1}{4}\cos 2x(2)$$
$$= \frac{1}{2}(1 - \cos 2x) = \sin^2 x$$

28. $y = \dfrac{1 + \sin x}{1 - \sin x}$

$$y' = \frac{(1 - \sin x)\cos x - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$$
$$= \frac{2\cos x}{(1 - \sin x)^2}$$

29. $y = \dfrac{2}{3}\sin^{3/2} x - \dfrac{2}{7}\sin^{7/2} x$

$$y' = \sin^{1/2} x \cos x - \sin^{5/2} x \cos x$$
$$= (\cos x)\sqrt{\sin x}(1 - \sin^2 x)$$
$$= (\cos^3 x)\sqrt{\sin x}$$

30. $y = \dfrac{\sec^7 x}{7} - \dfrac{\sec^5 x}{5}$

$$y' = \sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x)$$
$$= \sec^5 x \tan x(\sec^2 x - 1)$$
$$= \sec^5 x \tan^3 x$$

31. $y = -x \tan x$

$$y' = -x \sec^2 x - \tan x$$

32. $y = x \cos x - \sin x$

$$y' = -x \sin x + \cos x - \cos x = -x \sin x$$

33. $y = \dfrac{\sin x}{x^2}$

$$y' = \frac{(x^2)\cos x - (\sin x)(2x)}{x^4} = \frac{x \cos x - 2 \sin x}{x^3}$$

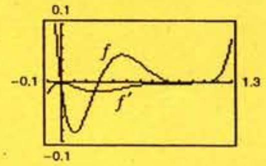
34. $y = \dfrac{\cos(x-1)}{x-1}$

$$y' = \frac{-(x-1)\sin(x-1) - \cos(x-1)(1)}{(x-1)^2}$$
$$= -\frac{1}{(x-1)^2}[(x-1)\sin(x-1) + \cos(x-1)]$$

35. $f(t) = t^2(t-1)^5$

$$f'(t) = t(t-1)^4(7t-2)$$

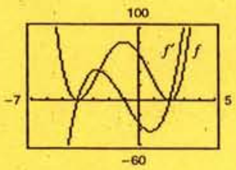
The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.



36. $f(x) = (x^2 + 2x - 8)^2$

$$f'(x) = 4(x^3 + 3x^2 - 6x - 8)$$
$$= 4(x-2)(x+1)(x+4)$$

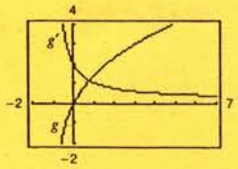
The zeros of f' correspond to the points on the graph of f where the tangent line is horizontal.



37. $g(x) = 2x(x+1)^{-1/2}$

$$g'(x) = \frac{x+2}{(x+1)^{3/2}}$$

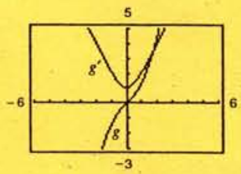
g' does not equal zero for any value of x in the domain. The graph of g has no horizontal tangent lines.



38. $g(x) = x(x^2+1)^{1/2}$

$$g'(x) = \frac{2x^2+1}{\sqrt{x^2+1}}$$

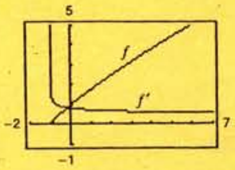
g' does not equal zero for any value of x. The graph of g has no horizontal tangent lines.



39. $f(t) = (t+1)^{1/2}(t+1)^{1/3} = (t+1)^{5/6}$

$$f'(t) = \frac{5}{6(t+1)^{1/6}}$$

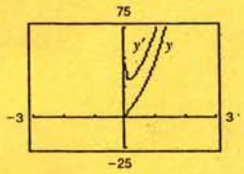
f' does not equal zero for any x in the domain. The graph of f has no horizontal tangent lines.



40. $y = \sqrt{3x}(x+2)^3$

$$y' = \frac{3(x+2)^2(7x+2)}{2\sqrt{3x}}$$

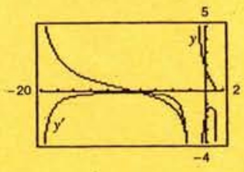
y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



41. $y = \tan\sqrt{1-x}$

$$y' = -\frac{\sec^2\sqrt{1-x}}{2\sqrt{1-x}}$$

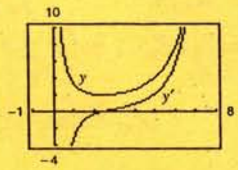
y' does not equal zero for any x in the domain. The graph has no horizontal tangent lines.



42. $y = 2\csc^3\sqrt{x}$

$$y' = -\frac{3}{\sqrt{x}}\csc^3\sqrt{x}\cot\sqrt{x}$$

The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



43. $f(x) = \cos\dfrac{\pi x}{2}$

$$f'(x) = -\frac{\pi}{2}\sin\frac{\pi x}{2}$$

$$f'\left(\frac{2}{3}\right) = -\frac{\pi}{2}\cdot\frac{\sqrt{3}}{2} = -\frac{\pi\sqrt{3}}{4}$$

44. $f(x) = x + 8x^{-2}$

$$f'(x) = 1 - 16x^{-3}$$

$$f'(2) = 1 - \frac{16}{2^3} = 1 - 2 = -1$$

45. $y = 2x^2 + \sin 2x$

$$y' = 4x + 2\cos 2x$$

$$y'' = 4 - 4\sin 2x$$

46. $y = x^{-1} + \tan x$

$$y' = -x^{-2} + \sec^2 x$$

$$y'' = 2x^{-3} + 2\sec x(\sec x\tan x)$$

$$= \frac{2}{x^3} + 2\sec^2 x\tan x$$

47. $f(x) = \cot x$

$$f'(x) = -\csc^2 x$$

$$f'' = -2\csc x(-\csc x\cdot\cot x)$$

$$= 2\csc^2 x\cot x$$

48. $y = \sin^2 x$

$$y' = 2\sin x\cos x = \sin 2x$$

$$y'' = 2\cos 2x$$

49. $f(t) = \dfrac{t}{(1-t)^2}$

$$f'(t) = \frac{t+1}{(1-t)^3}$$

$$f''(t) = \frac{2(t+2)}{(1-t)^4}$$

50. $g(x) = \dfrac{6x-5}{x^2+1}$

$$g'(x) = \frac{2(-3x^2+5x+3)}{(x^2+1)^2}$$

$$g''(x) = \frac{2(6x^3-15x^2-18x+5)}{(x^2+1)^3}$$

51. $g(x) = x \tan x$

$g'(x) = x \sec^2 x + \tan x$

$g''(x) = 2 \sec^2 x(x \tan x + 1)$

52. $h(x) = x\sqrt{x^2 - 1}$

$h'(x) = \dfrac{2x^2 - 1}{\sqrt{x^2 - 1}}$

$h''(x) = \dfrac{x(2x^2 - 3)}{(x^2 - 1)^{3/2}}$

53.
$$x^2 + 3xy + y^3 = 10$$
$$2x + 3xy' + 3y + 3y^2y' = 0$$
$$3(x + y^2)y' = -(2x + 3y)$$
$$y' = \frac{-(2x + 3y)}{3(x + y^2)}$$

54.
$$x^2 + 9y^2 - 4x + 3y = 0$$
$$2x + 18yy' - 4 + 3y' = 0$$
$$3(6y + 1)y' = 4 - 2x$$
$$y' = \frac{4 - 2x}{3(6y + 1)}$$

55.
$$y\sqrt{x} - x\sqrt{y} = 16$$
$$y\left(\frac{1}{2}x^{-1/2}\right) + x^{1/2}y' - x\left(\frac{1}{2}y^{-1/2}y'\right) - y^{1/2} = 0$$
$$\left(\sqrt{x} - \frac{x}{2\sqrt{y}}\right)y' = \sqrt{y} - \frac{y}{2\sqrt{x}}$$
$$\frac{2\sqrt{xy} - x}{2\sqrt{y}}y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}}$$
$$y' = \frac{2\sqrt{xy} - y}{2\sqrt{x}} \cdot \frac{2\sqrt{y}}{2\sqrt{xy} - x} = \frac{2y\sqrt{x} - y\sqrt{y}}{2x\sqrt{y} - x\sqrt{x}}$$

56.
$$y^2 = x^3 - x^2y + xy - y^2$$
$$0 = x^3 - x^2y + xy - 2y^2$$
$$0 = 3x^2 - x^2y' - 2xy + xy' + y - 4yy'$$
$$(x^2 - x + 4y)y' = 3x^2 - 2xy + y$$
$$y' = \frac{3x^2 - 2xy + y}{x^2 - x + 4y}$$

57.
$$x \sin y = y \cos x$$
$$(x \cos y)y' + \sin y = -y \sin x + y' \cos x$$
$$y'(x \cos y - \cos x) = -y \sin x - \sin y$$
$$y' = \frac{y \sin x + \sin y}{\cos x - x \cos y}$$

58.
$$\cos(x + y) = x$$
$$-(1 + y') \sin(x + y) = 1$$
$$-y' \sin(x + y) = 1 + \sin(x + y)$$
$$y' = -\frac{1 + \sin(x + y)}{\sin(x + y)}$$
$$= -\csc(x + 1) - 1$$

59. $y = (x + 3)^3$

$y' = 3(x + 3)^2$

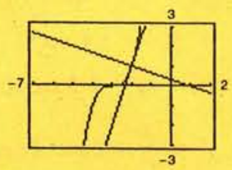
At $(-2, 1)$: $y' = 3$

Tangent line: $y - 1 = 3(x + 2)$

$3x - y + 7 = 0$

Normal line: $y - 1 = -\frac{1}{3}(x + 2)$

$x + 3y - 1 = 0$



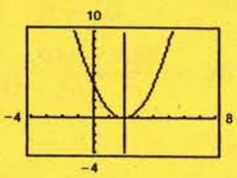
60. $y = (x - 2)^2$

$y' = 2(x - 2)$

At $(2, 0)$: $y' = 0$

Tangent line: $y = 0$

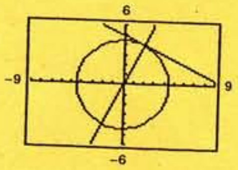
Normal line: $x = 2$



61. $x^2 + y^2 = 20$

$2x + 2yy' = 0$

$y' = -\dfrac{x}{y}$



At $(2, 4)$: $y' = -\dfrac{1}{2}$

Tangent line: $y - 4 = -\dfrac{1}{2}(x - 2)$

$x + 2y - 10 = 0$

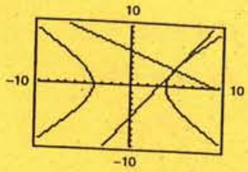
Normal line: $y - 4 = 2(x - 2)$

$2x - y = 0$

62. $x^2 - y^2 = 16$

$2x - 2yy' = 0$

$y' = \dfrac{x}{y}$



At $(5, 3)$: $y' = \dfrac{5}{3}$

Tangent line: $y - 3 = \dfrac{5}{3}(x - 5)$

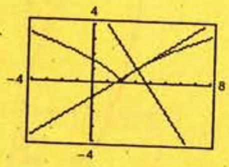
$5x - 3y - 16 = 0$

Normal line: $y - 3 = -\dfrac{3}{5}(x - 5)$

$3x + 5y - 30 = 0$

63. $y = (x - 2)^{2/3}$

$y' = \dfrac{2}{3}(x - 2)^{-1/3} = \dfrac{2}{3\sqrt[3]{x - 2}}$



At $(3, 1)$: $y' = \dfrac{2}{3}$

Tangent line: $y - 1 = \dfrac{2}{3}(x - 3)$

$2x - 3y - 3 = 0$

Normal line: $y - 1 = -\dfrac{3}{2}(x - 3)$

$3x + 2y - 11 = 0$

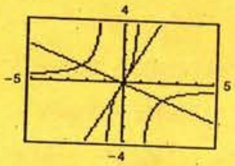
64. $y = \dfrac{2x}{1 - x^2}$

$y' = \dfrac{2(1 - x^2) - 2x(-2x)}{(1 - x^2)^2} = \dfrac{2(x^2 + 1)}{(1 - x^2)^2}$

At $(0, 0)$: $y' = 2$

Tangent line: $y = 2x$

Normal line: $y = -\dfrac{1}{2}x$



65. $f(x) = \dfrac{1}{3}x^3 + x^2 - x - 1$

$f'(x) = x^2 + 2x - 1$

(a) $x^2 + 2x - 1 = -1$

$x(x + 2) = 0$

$(0, -1), \left(-2, \dfrac{7}{3}\right)$

(b) $x^2 + 2x - 1 = 2$

$(x + 3)(x - 1) = 0$

$(-3, 2), \left(1, -\dfrac{2}{3}\right)$

(c) $x^2 + 2x - 1 = 0$

$(x + 1)^2 = 2$

$x = -1 \pm \sqrt{2}$

$\left(-1 + \sqrt{2}, \dfrac{2\left(1 - 2\sqrt{2}\right)}{3}\right)$,

$\left(-1 - \sqrt{2}, \dfrac{2\left(1 + 2\sqrt{2}\right)}{3}\right)$

66. $f(x) = x^2 + 1$

$f'(x) = 2x$

(a) $2x = -1$, $x = -\frac{1}{2}$, $\left(-\frac{1}{2}, \frac{5}{4}\right)$

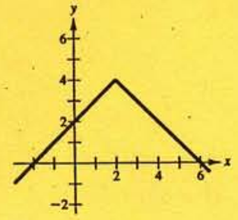
(b) $2x = 0$, $x = 0$, $(0, 1)$

(c) $2x = 1$, $x = \frac{1}{2}$, $\left(\frac{1}{2}, \frac{5}{4}\right)$

67. $f(x) = 4 - |x - 2|$

(a) Continuous at $x = 2$.

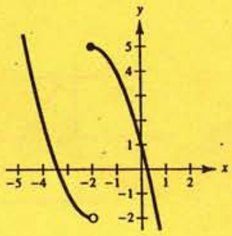
(b) Not differentiable at $x = 2$ because of the sharp turn in the graph.



68. $f(x) = \begin{cases} x^2 + 4x + 2, & \text{if } x < -2 \\ 1 - 4x - x^2, & \text{if } x \ge -2 \end{cases}$

(a) Nonremovable discontinuity at $x = -2$.

(b) Not differentiable at $x = -2$ because the function is discontinuous there.



69.
$$\begin{aligned} y &= 2\sin x + 3\cos x \\ y' &= 2\cos x - 3\sin x \\ y'' &= -2\sin x - 3\cos x \\ y'' + y &= -(2\sin x + 3\cos x) + (2\sin x + 3\cos x) \\ &= 0 \end{aligned}$$

70.
$$\begin{aligned} y &= \frac{(10 - \cos x)}{x} \\ xy + \cos x &= 10 \\ xy' + y - \sin x &= 0 \\ xy' &= \sin x - y \\ xy' + y &= (\sin x - y) + y = \sin x \end{aligned}$$

71. $T = 700(t^2 + 4t + 10)^{-1}$

$$T' = \frac{-1400(t + 2)}{(t^2 + 4t + 10)^2}$$

(a) When $t = 1$,

$$T' = \frac{-1400(1 + 2)}{(1 + 4 + 10)^2} \approx -18.667 \text{ deg/hr.}$$

(b) When $t = 3$,

$$T' = \frac{-1400(3 + 2)}{(9 + 12 + 10)^2} \approx -7.284 \text{ deg/hr.}$$

(c) When $t = 5$,

$$T' = \frac{-1400(5 + 2)}{(25 + 30 + 10)^2} \approx -3.240 \text{ deg/hr.}$$

(d) When $t = 10$,

$$T' = \frac{-1400(10 + 2)}{(100 + 40 + 10)^2} \approx -0.747 \text{ deg/hr.}$$

72. $v = \sqrt{2gh} = \sqrt{2(32)h} = 8\sqrt{h}$

$$\frac{dv}{dh} = \frac{4}{\sqrt{h}}$$

(a) When $h = 9$, $\frac{dv}{dh} = \frac{4}{3}$ ft/sec.

(b) When $h = 4$, $\frac{dv}{dh} = 2$ ft/sec.

73. $F = 200\sqrt{T}$

$$F'(T) = \frac{100}{\sqrt{T}}$$

(a) When $T = 4$, $F'(4) = 50$ vibrations/sec/lb.

(b) When $T = 9$, $F'(9) = 33\frac{1}{3}$ vibrations/sec/lb.

74. $s = -16t^2 + s_0$

First ball:

$$-16t^2 + 100 = 0$$

$$t = \sqrt{\frac{100}{16}} = \frac{10}{4} = 2.5 \text{ seconds to hit ground}$$

Second ball:

$$-16t^2 + 75 = 0$$

$$t^2 = \sqrt{\frac{75}{16}} = \frac{5\sqrt{3}}{4} \approx 2.165 \text{ seconds to hit ground}$$

Since the second ball was released one second after the first ball, the first ball will hit the ground first. The second ball will hit the ground $3.165 - 2.5 = 0.665$ second later.

75. Assume that the stone is thrown from an initial height of $s_0 = 0$. Thus, the position equation is $s = -16t^2 + v_0t$. The maximum value of s occurs when $ds/dt = 0$ and thus

$$\frac{ds}{dt} = -32t + v_0 = 0$$

$$-32t = -v_0$$

$$t = \frac{v_0}{32}.$$

This means that the maximum height is

$$s = -16\left(\frac{v_0}{32}\right)^2 + v_0\left(\frac{v_0}{32}\right) = \frac{v_0^2}{64}.$$

If s is to attain a value of 49, then

$$\frac{v_0^2}{64} = 49$$

$$v_0^2 = 3136$$

$$v_0 = 56 \text{ ft/sec.}$$

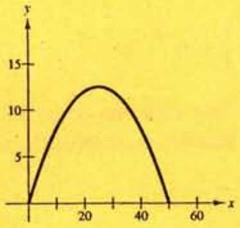
76. $s(t) = -16t^2 + 14{,}400 = 0$

$$16t^2 = 14{,}400$$

$$t = 30 \text{ sec}$$

Since 600 mph $= \frac{1}{6}$ mi/sec, in 30 seconds the bomb will move horizontally $\left(\frac{1}{6}\right)(30) = 5$ miles.

77. (a)



Total horizontal distance: 50

(b) $0 = x - 0.02x^2$

$$0 = x\left(x - \frac{x}{50}\right) \text{ implies } x = 50.$$

(c) Ball reaches maximum height when $x = 25$.

(d)

$$y = x - 0.02x^2$$

$$y' = 1 - 0.04x$$

$$y'(0) = 1$$

$$y'(10) = 0.6$$

$$y'(25) = 0$$

$$y'(30) = -0.2$$

$$y'(50) = -1$$

(e) $y'(25) = 0$

78. $y = \sqrt{x}$

$$\frac{dy}{dt} = 2 \text{ units/sec}$$

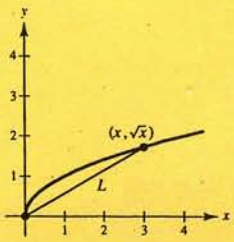
$$\frac{dy}{dt} = \frac{1}{2\sqrt{x}}\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 2\sqrt{x}\frac{dy}{dt} = 4\sqrt{x}$$

(a) When $x = \frac{1}{2}$, $\frac{dx}{dt} = 2\sqrt{2}$ units/sec.

(b) When $x = 1$, $\frac{dx}{dt} = 4$ units/sec.

(c) When $x = 4$, $\frac{dx}{dt} = 8$ units/sec.

79. $y = \sqrt{x}$



$L^2 = x^2 + y^2$

$\frac{dy}{dt} = 2$ units/sec

$L^2 = y^4 + y^2$

$$2L\frac{dL}{dt} = (4y^3 + 2y)\frac{dy}{dt}$$

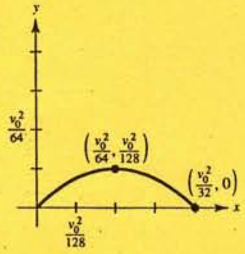
$$\frac{dL}{dt} = \frac{4y^3 + 2y}{2L}\frac{dy}{dt} = \frac{4y^3 + 2y}{L} = \frac{(4x + 2)\sqrt{x}}{L}$$

(a) When $x = \frac{1}{2}$, $L = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\sqrt{3}}{2}$ and $\frac{dL}{dt} = \frac{(2 + 2)(1/\sqrt{2})}{\sqrt{3}/2} = \frac{8}{\sqrt{6}} = \frac{4\sqrt{6}}{3}$ units/sec.

(b) When $x = 1$, $L = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$ and $\frac{dL}{dt} = \frac{(4 + 2)(1)}{\sqrt{2}} = 3\sqrt{2}$ units/sec.

(c) When $x = 4$, $L = \sqrt{(4)^2 + (2)^2} = 2\sqrt{5}$ and $\frac{dL}{dt} = \frac{(16 + 2)(2)}{2\sqrt{5}} = \frac{18}{\sqrt{5}} = \frac{18\sqrt{5}}{5}$ units/sec.

80.



(a) $y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right)$

$= 0$ if $x = 0$ or $x = \frac{v_0^2}{32}$.

Projectile strikes the ground when $x = v_0^2/32$.

Projectile reaches its maximum height at $x = v_0^2/64$. (one-half the distance)

(b) $y' = 1 - \frac{64}{v_0^2}x$

When $x = \frac{v_0^2}{64}$, $y' = 1 - \frac{64}{v_0^2}\left(\frac{v_0^2}{64}\right) = 0$.

(d) $v_0 = 70$ ft/sec

Range: $x = \frac{v_0^2}{32} = \frac{(70)^2}{32} = 153.125$ ft

Maximum height: $y = \frac{v_0^2}{128} = \frac{(70)^2}{128} \approx 38.28$ ft

(c) $y = x - \frac{32}{v_0^2}x^2 = x\left(1 - \frac{32}{v_0^2}x\right) = 0$

when $x = 0$ and $x = x_0^2/32$. Therefore, the range is $x = v_0^2/32$. When the initial velocity is doubled the range is

$$x = \frac{(2v_0)^2}{32} = \frac{4v_0^2}{32}$$

or four times the initial range. From part (a), the maximum height occurs when $x = v_0^2/64$. The maximum height is

$$y\left(\frac{v_0^2}{64}\right) = \frac{v_0^2}{64} - \frac{32}{v_0^2}\left(\frac{v_0^2}{64}\right)^2 = \frac{v_0^2}{64} - \frac{v_0^2}{128} = \frac{v_0^2}{128}.$$

If the initial velocity is doubled, the maximum height is

$$y\left[\frac{(2v_0)^2}{64}\right] = \frac{(2v_0)^2}{128} = 4\left(\frac{v_0^2}{128}\right)$$

or four times the original maximum height.