Integrate each function.

1a) $\displaystyle\int x^2 e^{x^3}\, dx$

Let $u = x^3$, $du = 3x^2\, dx$

$= \frac{1}{3}\int e^{x^3}\left(3x^2\right) dx = \frac{1}{3}e^{x^3} + C$

1b) $\displaystyle\int e^{2x-1}\, dx$

1c) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

1d) $\displaystyle\int_0^{1/2} e^{\sin \pi x} \cos \pi x\, dx$

1e) $\displaystyle\int_1^3 \frac{e^{3/x}}{x^2}\, dx$

1f) $\displaystyle\int \frac{5 - e^x}{e^{2x}}\, dx$

2a) $\displaystyle\int \frac{e^{-x}}{1 + e^{-x}}\, dx$

2b) $\displaystyle\int e^x\sqrt{1 - e^x}\, dx$

2c) $\displaystyle\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$

Let $u = 1 + e^{-x}$, $du = -e^{-x}\, dx$

$= -\int \frac{-e^{-x}}{1 + e^{-x}}\, dx = -\ln\left(1 + e^{-x}\right) + C$

$= \ln\left(\frac{e^x}{e^x + 1}\right) + C$

$= x - \ln\left(e^x + 1\right) + C$

2d) $\displaystyle\int_0^3 \frac{2e^{2x}}{1 + e^{2x}}\,dx$

2e) $\displaystyle\int e^{-x}\tan(e^{-x})\,dx$

3a) $\displaystyle\int x(5^{-x^2})\,dx$

$$= -\frac{1}{2}\int 5^{-x^2}(-2x)\,dx$$

$$= -\left(\frac{1}{2}\right)\frac{5^{-x^2}}{\ln 5} + C$$

$$= \frac{-1}{2\ln 5}\left(5^{-x^2}\right) + C$$

3b) $\displaystyle\int (x^2 + 2^{-x})\,dx$

3c) $\displaystyle\int \frac{3^{2x}}{1 + 3^{2x}}\,dx$

Solve the differential equation with the given solution.

4a) $\dfrac{dy}{dx} = 2e^{-x/2}, \quad (0, 1)$

$$y = \int 2e^{-x/2}\,dx = -4\int e^{-x/2}\left(-\frac{1}{2}\,dx\right)$$

$$= -4e^{-x/2} + C$$

$$(0, 1):\ 1 = -4e^0 + C = -4 + C \Rightarrow C = 5$$

$$y = -4e^{-x/2} + 5$$

4b) $\dfrac{dy}{dx} = 0.4^{x/3}, \quad \left(0, \tfrac{1}{2}\right)$

5) AP MULTIPLE CHOICE EXAMPLES

1) $\frac{1}{2}\int e^{\frac{t}{2}}\,dt =$

(A) $e^{-t}+C$ (B) $e^{-\frac{t}{2}}+C$ (C) $e^{\frac{t}{2}}+C$ (D) $2e^{\frac{t}{2}}+C$ (E) $e^{t}+C$

2) $\int_0^1 x^3 e^{x^4}\,dx =$

(A) $\frac{1}{4}(e-1)$ (B) $\frac{1}{4}e$ (C) $e-1$ (D) e (E) $4(e-1)$

3) $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$ is

(A) 0 (B) 1 (C) $e-1$ (D) e (E) $e+1$

4) A particle with velocity at any time t given by $v(t)=e^t$ moves in a straight line. How far does the particle move from $t=0$ to $t=2$?

(A) e^2-1 (B) $e-1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

5) $\int \frac{x^2}{e^{x^3}}\,dx =$

(A) $-\frac{1}{3}\ln e^{x^3} + C$ (B) $-\frac{e^{x^3}}{3} + C$ (C) $-\frac{1}{3e^{x^3}} + C$

(D) $\frac{1}{3}\ln e^{x^3} + C$ (E) $\frac{x^3}{3e^{x^3}} + C$

HINT: Need to use substitution TWICE here!

6) The number of bacteria in a culture is growing at a rate of $3000e^{\frac{2t}{5}}$ per unit of time t. At $t = 0$, the number of bacteria present was 7,500. Find the number present at $t = 5$.

(A) $1,200e^2$ (B) $3,000e^2$ (C) $7,500e^2$ (D) $7,500e^5$ (E) $\frac{15,000}{7}e^7$

7) If the graph of $y = f(x)$ contains the point $(0, 2)$, $\frac{dy}{dx} = \frac{-x}{ye^{x^2}}$ and $f(x) > 0$ for all x, then $f(x) =$

(A) $3+e^{-x^2}$ (B) $\sqrt{3} + e^{-x}$ (C) $1 + e^{-x}$

(D) $\sqrt{3+e^{-x^2}}$ (E) $\sqrt{3+e^{x^2}}$