Evaluate each integral without a calculator.

1a) $\int_{-1}^{1} (t^2 - 2)\, dt$

$= \left[\frac{t^3}{3} - 2t\right]_{-1}^{1}$

$= \left(\frac{1}{3} - 2\right) - \left(-\frac{1}{3} + 2\right) = -\frac{10}{3}$

1b) $\int_{-1}^{0} (2x - 1)\, dx$

$x^2 - x \Big|_{-1}^{0}$

$(0-0) - (1-(-1))$

$0 - 2$

-2

1c) $\int_{-1}^{0} \left(t^{1/3} - t^{2/3}\right) dt$

$\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3} \Big|_{-1}^{0}$

$(0-0) - \left(\frac{3}{4} - \left(\frac{-3}{5}\right)\right)$

$0 - \left(\frac{3}{4} + \frac{3}{5}\right)$

$-27/20$

2a) $\int_{0}^{1} (2t - 1)^2\, dt$

$= \int_0^1 \left(4t^2 - 4t + 1\right) dt$

$= \left[\frac{4}{3}t^3 - 2t^2 + t\right]_0^1 = \frac{4}{3} - 2 + 1 = \frac{1}{3}$

2b) $\int_{1}^{2} \left(\frac{3}{x^2} - 1\right) dx$

$\int_1^2 (3x^{-2} - 1)\, dx$

$-3x^{-1} - x \Big|_1^2$

$-\frac{3}{x} - x \Big|_1^2$

$\left(-\frac{3}{2} - 2\right) - (-3 - 1)$

$-\frac{7}{2} + (+4)$

$\frac{1}{2}$

2c) $\int_{1}^{4} \frac{u - 2}{\sqrt{u}}\, du = \int_1^4 \frac{u}{u^{1/2}} - \frac{2}{u^{1/2}}\, du$

$\int_1^4 u^{1/2} - 2u^{-1/2}\, du$

$\frac{2}{3}u^{3/2} - 4u^{1/2} \Big|_1^4$

$\left(\frac{16}{3} - 8\right) - \left(\frac{2}{3} - 4\right)$

$-\frac{8}{3} - \left(\frac{-10}{3}\right)$

$\frac{2}{3}$

3a) $\int_{-\pi/6}^{\pi/6} \sec^2 x\, dx$

$= [\tan x]_{-\pi/6}^{\pi/6}$

$= \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}$

3b) $\int_{0}^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\, d\theta$

$\int_0^{\pi/4} \frac{\cos^2\theta}{\cos^2\theta}\, d\theta$

$\int_0^{\pi/4} 1\, d\theta$

$\theta \Big|_0^{\pi/4}$

$\frac{\pi}{4} - 0$

$\frac{\pi}{4}$

3c) $\int_{-\pi/3}^{\pi/3} 4 \sec\theta \tan\theta\, d\theta$

$4\int_{-\pi/3}^{\pi/3} \sec\theta \tan\theta\, d\theta$

$4\left[\sec\theta \Big|_{-\pi/3}^{\pi/3}\right]$

$4\left[\sec\frac{\pi}{3} - \sec\left(\frac{-\pi}{3}\right)\right]$

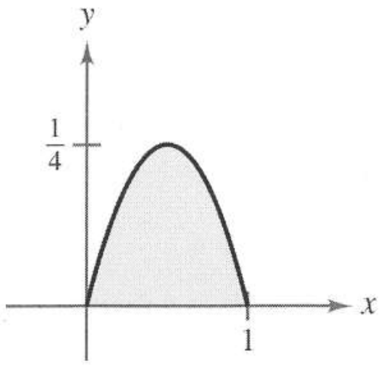
$4[2 - 2]$

$4 \cdot 0$

0

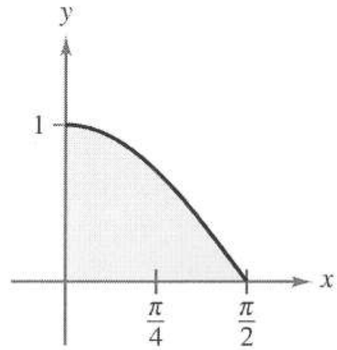
Determine the area of the shaded region without a calculator.

4a) $y = x - x^2$



$$A = \int_0^1 \left(x - x^2\right) dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$$

4b) $y = \cos x$



$$A = \int_0^{\frac{\pi}{2}} \cos x \, dx$$
$$= \sin x \Big|_0^{\frac{\pi}{2}} = \sin\tfrac{\pi}{2} - \sin 0 = 1 - 0 = \boxed{1}$$

Use the fundamental theorem of calculus to find and expression $(F(x))$ for each integral. Then find its derivative $F'(x)$ thus demonstrating the 2nd Fundamental Theorem of Calculus.

5a) $F(x) = \displaystyle\int_8^x \sqrt[3]{t}\, dt$

$$F(x) = \int_8^x \sqrt[3]{t}\, dt = \left[\frac{3}{4}t^{4/3}\right]_8^x = \frac{3}{4}\left(x^{4/3} - 16\right) = \frac{3}{4}x^{4/3} - 12$$

$$F'(x) = \frac{d}{dx}\left[\frac{3}{4}x^{4/3} - 12\right] = x^{1/3} = \sqrt[3]{x}$$

5b) $F(x) = \displaystyle\int_{\pi/4}^{x} \sec^2 t\, dt$

$F(x) = \tan \Big|_{\pi/4}^{x}$

$F(x) = \tan x - \tan\frac{\pi}{4}$

$F(x) = \tan x - 1$

$F'(x) = \sec^2 x - 0$

$= \sec^2 x$

5c) $F(x) = \displaystyle\int_{x}^{x+2} (4t + 1)\, dt$

$F(x) = 2t^2 + t \Big|_x^{x+2}$

$F(x) = \left(2(x+2)^2 + (x+2)\right) - \left(2x^2 + x\right)$

$F'(x) = \left(4(x+2)\cdot 1 + 1\right) - \left(4x + 1\right)$

$= 4(x+2) - 4x$

$= 8$

Use the 2nd Fundamental Theorem of Calculus to find $F'(x)$.

6a) $F(x) = \int_{-1}^{x} \sqrt{t^4 + 1}\, dt$

$F'(x) = \sqrt{x^4 + 1}$

6b) $F(x) = \int_{0}^{x} t \cos t\, dt$

$F'(x) = x \cdot \cos x$

6c) $F(x) = \int_{1}^{x} \frac{t^2}{t^2 + 1}\, dt$

$F'(x) = \frac{x^2}{x^2 + 1}$

7a) $F(x) = \int_{3}^{\tan x} \sqrt{2t - 1}\, dt$

$F'(x) = \sec^2 x \bullet \sqrt{2 \tan x - 1}$

7b) $F(x) = \int_{3}^{e^x} \log(t - 2)\, dt$

$F'(x) = \log(e^x - 2) \cdot e^x$

7c) $F(x) = \int_{x}^{3x^2} \cot(t)\, dt$

$F'(x) = 6x \cdot \cot(3x^2) - \cot x$

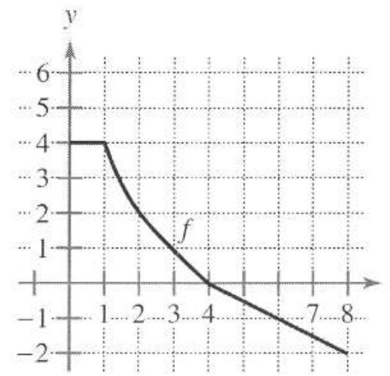
8a) Let $g(x) = \int_0^x f(t)\, dt$, where f is the function whose graph is shown in the figure.

(a) Estimate $g(0)$, $g(2)$, $g(4)$, $g(6)$, and $g(8)$.

(b) Find the largest open interval on which g is increasing. Find the largest open interval on which g is decreasing.

(c) Identify any extrema of g.

(d) Sketch a rough graph of g.



$g(x) = \int_0^x f(t)\, dt$

(a) $g(0) = \int_0^0 f(t)\, dt = 0$

$g(2) = \int_0^2 f(t)\, dt \approx 4 + 2 + 1 = 7$

$g(4) = \int_0^4 f(t)\, dt \approx 7 + 2 = 9$

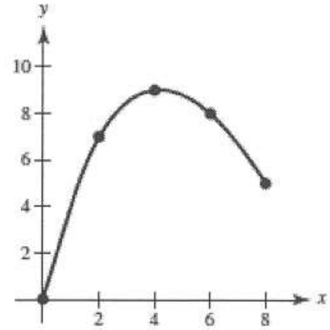
$g(6) = \int_0^6 f(t)\, dt \approx 9 + (-1) = 8$

$g(8) = \int_0^8 f(t)\, dt \approx 8 - 3 = 5$

(b) g increasing on $(0, 4)$ and decreasing on $(4, 8)$

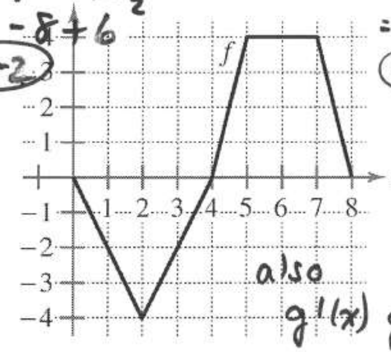
(c) g is a maximum of 9 at $x = 4$.

(d)



a) $g(0)=\int_0^0 f(t)\,dt$ $= 0$ $g(2)=\int_0^2 f(t)\,dt$ $= -\left(\frac{2\cdot 4}{2}\right)$ $= -4$ $g(4)=\int_0^4 f(t)\,dt$ $= -\left(\frac{4\cdot 4}{2}\right)$ $= -8$ $g(6)=\int_0^6 f(t)\,dt$ $= \int_0^4 f(t)\,dt + \int_4^6 f(t)\,dt$ $= -8 + \frac{(2+1)\cdot 4}{2}$ $= -8+6$ $= -2$ $g(8)=\int_0^8 f(t)\,dt$ $= \int_0^4 f(t)\,dt + \int_4^8 f(t)\,dt$ $= -8 + \frac{(4+2)\cdot 4}{2}$ $= -8+12$ $= 4$

8b) Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown in the figure.



also $g'(x)$ graph

(a) Estimate $g(0)$, $g(2)$, $g(4)$, $g(6)$, and $g(8)$.

(b) Find the largest open interval on which g is increasing. Find the largest open interval on which g is decreasing.

(c) Identify any extrema of g.

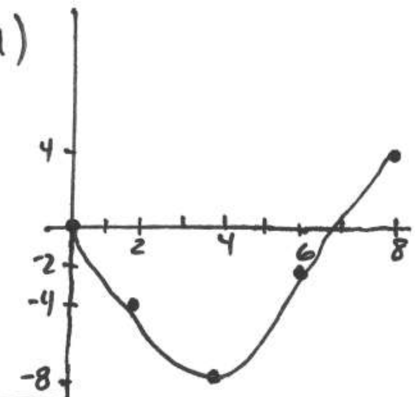
(d) Sketch a rough graph of g.

b) $g'(x) = \frac{d}{dx}\int_0^x f(t)$
$g'(x) = f(x)$

INC $(4,8)$
DEC $(0,4)$

c) rel MIN at $x=4$ so $(4,-8)$

d)



9) AP MULTIPLE CHOICE EXAMPLES

1) $\int_1^2 x^{-3}\,dx =$ $\left.\frac{x^{-2}}{-2}\right|_1^2 = \left.\frac{1}{-2x^2}\right|_1^2 = \frac{-1}{8} - \left(\frac{-1}{2}\right)$

(A) $-\frac{7}{8}$ (B) $-\frac{3}{4}$ (C) $\frac{15}{64}$ (D) $\frac{3}{8}$ (E) $\frac{15}{16}$

$-\frac{1}{8} + \frac{1}{2} = \frac{-2+8}{16} = \frac{6}{16} = \frac{3}{8}$

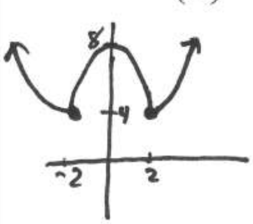
2) $\int_0^{\pi/4} \tan^2 x\,dx =$ $\int_0^{\pi/4} \sec^2 x - 1\,dx$

(A) $\frac{\pi}{4}-1$ (B) $1-\frac{\pi}{4}$ (C) $\frac{1}{3}$ (D) $\sqrt{2}-1$ (E) $\frac{\pi}{4}+1$

$\left.\tan x - x\right|_0^{\pi/4}$
$= \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$
$\left(1-\frac{\pi}{4}\right) - 0$

3) If $\begin{cases} f(x) = 8 - x^2 & \text{for } -2 \le x \le 2, \\ f(x) = x^2 & \text{elsewhere}, \end{cases}$ then $\int_{-1}^{3} f(x)\,dx$ is a number between

(A) 0 and 8 (B) 8 and 16 (C) 16 and 24 (D) 24 and 32 (E) 32 and 40



$\int_{-1}^2 f(x)\,dx + \int_2^3 f(x)\,dx$
$\int_{-1}^2 8-x^2\,dx + \int_2^3 x^2\,dx$
$\left.8x - \frac{1}{3}x^3\right|_{-1}^2 + \left.\frac{1}{3}x^3\right|_2^3$
$\left(16-\frac{8}{3}\right) - \left(-8+\frac{1}{3}\right) + \left(9-\frac{8}{3}\right)$

$16 - \frac{8}{3} + 8 - \frac{1}{3} + 9 - \frac{8}{3}$
$33 - \frac{17}{3}$
$33 - 5\frac{2}{3}$

4) $\int_0^1 \sqrt{x^2-2x+1}\,dx$ is $= \int_0^1 \sqrt{(x-1)^2}\,dx$

(A) -1 $= \int_0^1 x-1\,dx$

(B) $-\frac{1}{2}$ $= \frac{x^2}{2} - x \Big|_0^1$

(C) $\frac{1}{2}$ $= \left(\frac{1}{2} - 1\right) - (0 - 0)$

(D) 1 $= -\frac{1}{2}$

(E) none of the above

5) If $\int_{-2}^{2}\left(x^7+k\right)dx = 16$, then $k =$ $\frac{x^8}{8} + kx \Big|_{-2}^{2} = 16$

(A) -12 (B) -4 (C) 0 (D) 4 (E) 12

$\left(\frac{2^8}{8} + 2k\right) - \left(\frac{(-2)^8}{8} - 2k\right) = 16$

$\cancel{\frac{2^8}{8}} - \cancel{\frac{2^8}{8}} + 2k + 2k = 16$

$4k = 16$ $k = 4$

6) If the function f has a continuous derivative on $[0,c]$, then $\int_0^c f'(x)\,dx =$ $f(x)\Big|_0^c = f(c) - f(0)$

(A) $f(c)-f(0)$ (B) $|f(c)-f(0)|$ (C) $f(c)$ (D) $f(x)+c$ (E) $f''(c)-f''(0)$

7) If n is a known positive integer, for what value of k is $\int_1^k x^{n-1}\,dx = \frac{1}{n}$?

(A) 0 (B) $\left(\frac{2}{n}\right)^{1/n}$ (C) $\left(\frac{2n-1}{n}\right)^{1/n}$

(D) $2^{1/n}$ (E) 2^n

$\int_1^k x^{n-1}\,dx = \frac{1}{n}$

$\frac{x^{n-1+1}}{n}\Big|_1^k = \frac{1}{n}$

$\frac{x^n}{n}\Big|_1^k = \frac{1}{n}$

$\frac{k^n}{n} - \frac{1^n}{n} = \frac{1}{n}$

$n\left(\frac{k^n}{n} - \frac{1^n}{n}\right) = \frac{1}{n} \cdot n$

$k^n - 1 = 1$

$k^n = 2$

$k = 2^{1/n}$

8) If $F(x)=\int_0^x e^{-t^2}\,dt$, then $F'(x)=$ $e^{-x^2}\cdot 1$

(A) $2xe^{-x^2}$ (B) $-2xe^{-x^2}$ (C) $\dfrac{e^{-x^2+1}}{-x^2+1}-e$

(D) $e^{-x^2}-1$ (E) e^{-x^2}

9) For all $x>1$, if $f(x)=\int_1^x \frac{1}{t}\,dt$, then $f'(x)=$ $\frac{d}{dx}\int_1^x \frac{1}{t}\,dt = \frac{1}{x}$

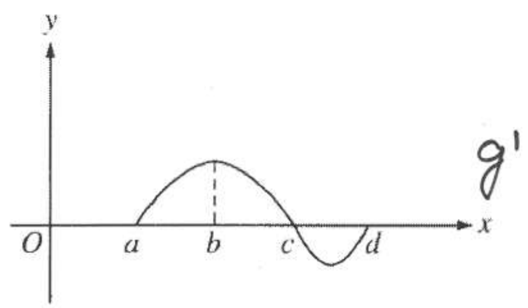
(A) 1 (B) $\frac{1}{x}$ (C) $\ln x-1$ (D) $\ln x$ (E) e^x

10) If $F(x)=\int_1^{x^2}\sqrt{1+t^3}\,dt$, then $F'(x)=$ $\frac{d}{dx}\int_1^{x^2}\sqrt{1+t^3}\,dt = \sqrt{1+(x^2)^3}\cdot 2x$

(A) $2x\sqrt{1+x^6}$ (B) $2x\sqrt{1+x^3}$ (C) $\sqrt{1+x^6}$

(D) $\sqrt{1+x^3}$ (E) $\int_1^{x^2}\dfrac{3t^2}{2\sqrt{1+t^3}}\,dt$

11)



The graph of f is shown in the figure above. If $g(x)=\int_a^x f(t)\,dt$, for what value of x does $g(x)$ have a maximum?

g': $+$ | c | $-$

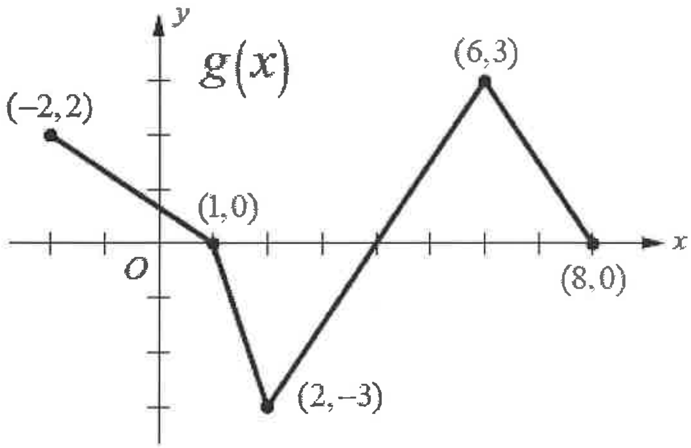
g inc　　g dec

$g'(x) = \frac{d}{dx}\int_a^x f(t)\,dt$

$g'(x) = f(x)$

(A) a
(B) b
(C) c
(D) d
(E) It cannot be determined from the information given.

12)



Given the graph of g(x) above, evaluate $\int_2^6 g'(x)\,dx$

A) 0 B) 6 C) 3 D) 1.5

NOTE: YOU DO NOT FIND AREA SINCE GRAPH GIVEN IS <u>NOT</u> $g'(x)$

$g(x)\Big|_2^6$

$g(6)-g(2)$

$3-(-3)$

6