Implicit Differentiation

Graph $x^2+y^2=25$ and estimate the slope of the tangent line at x = 0, and $x=\pm 5$.

When finding the derivative of NON-FUNCTIONS, it is **not** necessary to solve for y. In fact, sometimes it is impossible to solve for y though we can still find an expression for the derivative.

Find $\frac{dy}{dx}$ if $y=x\bullet\sin^2 x$

We may only differentiate with respect to **one variable**(usually it has been "x"). So, when differentiating a variable other than x requires us to differentiate as a composite function using $\frac{dy}{dx}$ as the symbolic form for the derivative of the inner function.

Find $\frac{dy}{dx}$ if $y=x\bullet y^2$ using the example above as a model.

$\frac{dy}{dx}$ = $x\bullet 2y\frac{dy}{dx}+y^2\bullet 1$

Needed because I do not know what y is in terms of x so this is how we say the derivative of the inner function "in terms of x"

Now solve for $\frac{dy}{dx}$

$$\frac{dy}{dx}-x\bullet 2y\frac{dy}{dx}=y^2$$

$$\frac{dy}{dx}(1-x\bullet 2y)=y^2$$

$$\frac{dy}{dx}=\frac{y^2}{(1-2xy)}$$

Note: This derivative needs both an x and y coordinate to find a slope(for the tangent line) of the graph anywhere.

EX) Find the slope of the graph of $x^2+y^2=25$ at x = 3 first by solving for y and using the chain rule and secondly by using implicit differentiation.

EX) Find $\frac{dy}{dx}$ for the equation $y^2 = 4x-3x^2y-6y$

Normal Line: Line **perpendicular** to the tangent line.

EX) Find the *equation* of the normal line to the graph of $x^2+y^2=25$ at x = 3.

A derivative can be written in terms of ANY VARIABLE using substitution

Remember, $\frac{d^2y}{dx^2}$ means the Second Derivative of y with respect to x

EX) If $y=3x^4$ find $\frac{d^2y}{dx^2}$ in terms of y.

$$y=3x^4$$

$$\frac{dy}{dx}=12x^3$$

$$\frac{d^2y}{dx^2}=36x^2 \quad \text{(in terms of } x\text{)}$$

or

$$\frac{d^2y}{dx^2}=36\sqrt{y/3} \quad \text{(in terms of } y\text{)}$$

Original equation solved for "x^2" and substituted in its place.

Substitution may also be needed when finding a second derivative implicitly.

EX) If $\frac{dy}{dx}=\frac{x}{y}$, find $\frac{d^2y}{dx^2}$