AP Calculus  Inc/Dec Functions & the 1st Derivative Test  Critical Homework

**Find the interval(s) on which the function is increasing or decreasing, and identify all relative extrema.**

1a)

$f(x) = x^2 - 4x$

(a) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

Critical number: $x = 2$

(b)

| Test intervals: | $-\infty < x < 2$ | $2 < x < \infty$ |
|---|---|---|
| Sign of $f'$: | $f' < 0$ | $f' > 0$ |
| Conclusion: | Decreasing | Increasing |

Decreasing on: $(-\infty, 2]$

Increasing on: $[2, \infty)$

(c) Relative minimum: $(2, -4)$

1b) $f(x) = \dfrac{x^5 - 5x}{5}$

$f(x) = \frac{1}{5}x^5 - x$

$f'(x) = x^4 - 1$

$0 = x^4 - 1$

$1 = x^4$

$\pm\sqrt[4]{1} = x$

$\pm 1 = x$

$f'(-2) = $ POS

$f'(0) = $ NEG

$f'(2) = $ POS

+ − +

$-1$ $1$

$\boxed{\text{INC } (-\infty, -1] \cup [1, \infty) \quad \text{DEC } [-1, 1]}$

relative MAX at $x = -1$

$f(-1) = 4/5 \rightarrow \boxed{(-1, 4/5)}$

relative MIN at $x = 1$

$f(1) = -4/5 \rightarrow \boxed{(1, -4/5)}$

1c) $f(x) = (x - 1)^2(x + 3)$

$f'(x) = (x-1)^2 \cdot 1 + (x+3) \cdot 2(x-1)' \cdot 1$

$0 = (x-1)^2 + 2(x+3)(x-1)$

$0 = (x-1)((x-1) + 2(x+3))$

$0 = (x-1)(3x+5)$

critical points $x = 1$ & $-5/3$

+ − +

$-5/3$ $1$

$f'(-3) = $ POS  $f'(0) = $ NEG  $f'(2) = $ POS

$\boxed{\text{INC } (-\infty, -5/3] \cup [1, \infty) \quad \text{DEC } [-5/3, 1]}$

rel max at $x = -5/3$  $f(-5/3) = \frac{256}{27}$

$\rightarrow \boxed{(-5/3, \frac{256}{27})}$

rel min at $x = 1$  $f(1) = 0$

$\rightarrow \boxed{(1, 0)}$

1d) $f(x) = \dfrac{x^2}{x^2 - 9}$

$f'(x) = \dfrac{(x^2-9) \cdot 2x - x^2 \cdot 2x}{(x^2-9)^2}$

$f'(x) = \dfrac{-18x}{(x^2-9)^2}$

$f'(x)$ undef. at $x = \pm 3$

$0 = \dfrac{-18x}{(x^2-9)^2}$

$0 = -18x$

$0 = x$

+ + − −

$-3$ $0$ $3$

$f'(-4) = $ POS  $f'(-2) = $ POS

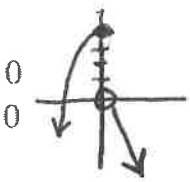
$f'(2) = $ NEG  $f'(4) = $ NEG

$\boxed{\text{INC } (-\infty, -3) \cup (-3, 0] \quad \text{DEC } [0, 3) \cup (3, \infty)}$

rel max at $x = 0$

$f(0) = 0 \rightarrow \boxed{(0, 0)}$

1e) $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$



$f'(x) = \begin{cases} -2x & x < 0 \\ -2 & x > 0 \end{cases}$

$f'(x)$ undefined at $x = 0$ and never equals zero

+ −

$0$

$f'(-1) = $ POS  $f'(1) = $ NEG

$\boxed{\text{INC } (-\infty, 0] \quad \text{DEC } (0, \infty)}$

relative MAX at $x = 0$

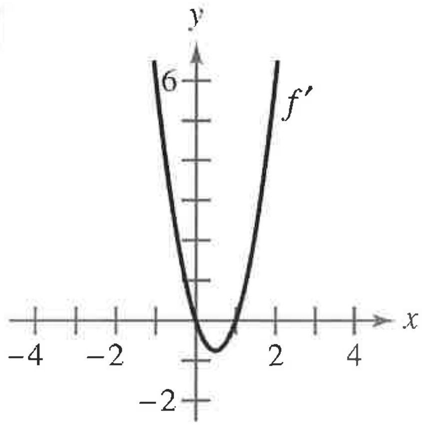
$f(0) = 4$

$\boxed{(0, 4)}$

NOTE: ALSO AN ABSOLUTE MAX

**Use the graph of $f'(x)$ to (a) identify the interval(s) on which $f(x)$ is increasing or decreasing, and (b) estimate the value(s) of $x$ at which $f(x)$ has a relative maximum or minimum.**

2a)



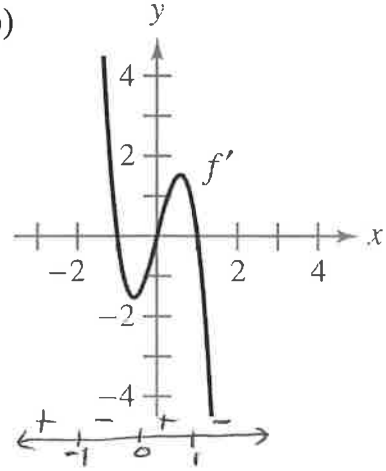

(a) Intervals of Increase $(-\infty, 0] \cup [1, \infty)$

Intervals of Decrease $[0, 1]$

(b) Relative maximum: $x = 0$

Relative minimum: $x = 1$

2b)



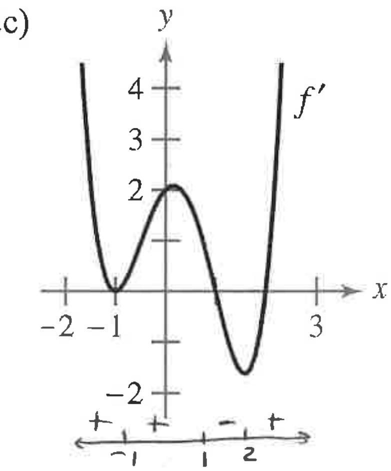

INC $(-\infty, -1] \cup [0, 1]$

DEC $[-1, 0] \cup [1, \infty)$

rel min at $x=0$

rel max at $x=-1$ & $x=1$

2c)




INC $(-\infty, 1] \cup [2, \infty)$

DEC $[1, 2]$

rel. min at $x = 2$

rel. max at $x = 1$

NOTE: a plateau occurs on $f(x)$ at $x = -1$

3) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x) = x + \dfrac{1}{x}$, then the set of values for which $f$ increases is

$= x + x^{-1}$

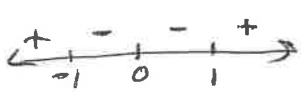
(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

$f'(x) = 1 - x^{-2}$

$f'(x) = 1 - \dfrac{1}{x^2}$

$f'(x)$ undefined at $x = 0$




$0 = 1 - \dfrac{1}{x^2}$

$0 = x^2 - 1$

$0 = (x+1)(x-1)$

NOTE:

$f'(x) = \dfrac{x^2 - 1}{x^2}$

also!

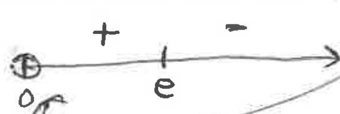
EASIER TO TEST $f'(x)$ VALUES

2) A point moves on the $x$-axis in such a way that its velocity at time $t$ $(t > 0)$ is given by $v = \dfrac{\ln t}{t}$.

At what value of $t$ does $v$ attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) $e$ (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for $v$.




$0 = \dfrac{1 - \ln t}{t^2}$

$0 = 1 - \ln t$

$\ln t = 1$

$e^1 = t$

$v = \dfrac{\ln t}{t}$

$v' = \dfrac{t \cdot \frac{1}{t} - \ln t}{t^2}$

$v' = \dfrac{1 - \ln t}{t^2}$

$v'(1)$ is POS $v'(e^2)$ is NEG

3) For what value of $k$ will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) −4 (B) −2 (C) 2 (D) 4 (E) None of these

$\frac{d}{dx}(x+kx^{-1}) = 1-kx^{-2}$

$\hookrightarrow 1-\frac{k}{x^2}=0$

$x^2-k=0$

$x^2=k$

$(-2)^2=k$

$+ \quad -$

$-2$

$1-\frac{4}{x^2}$ pos when $x=-3$

$1-\frac{4}{x^2}$ NEG when $x=-1$

so... rel max at $x=-2$

4) At $x=0$, which of the following is true of the function $f$ defined by $f(x)=x^2+e^{-2x}$?

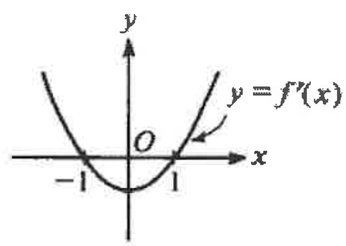
(A) $f$ is increasing.
(B) $f$ is decreasing.
(C) $f$ is discontinuous.
(D) $f$ has a relative minimum.
(E) $f$ has a relative maximum.

$f'(x) = 2x-2e^{-2x}$

$f'(0) = -2$

so... decreasing

5)




$+ \quad - \quad +$

$-1 \quad 1$

so...

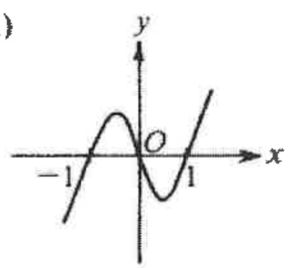
$f(x)$

INC $(-\infty,-1]\cup[1,\infty)$

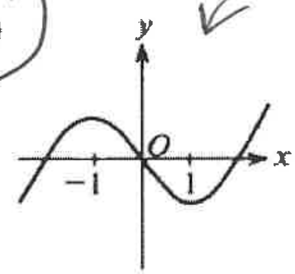
DEC $[-1,1]$

The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$?

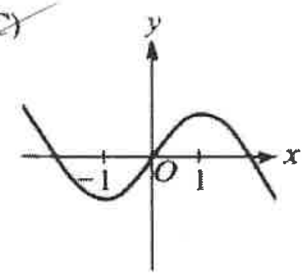
(A)



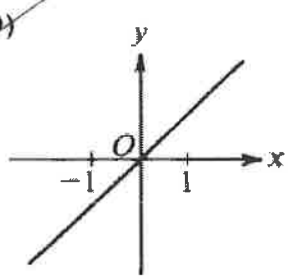

(B)



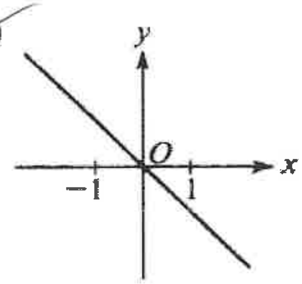

(C)



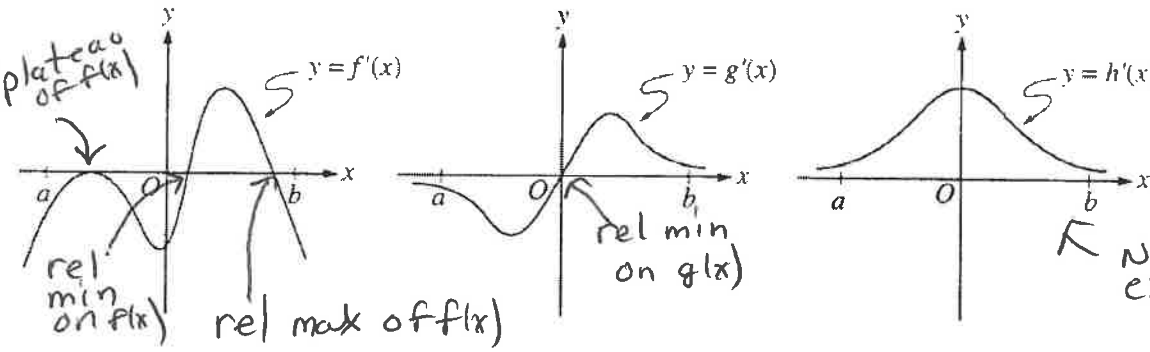

(D)




(E)



6) plateau of f(x)




rel min on f(x) rel max of f(x) rel min on g(x) No relative extrema of h(x)

The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

(A) $f$ only
(B) $g$ only
(C) $h$ only
(D) $f$ and $g$ only
(E) $f$, $g$, and $h$

MAX occurs when f(x) goes from INC to DEC... so f'(x) goes from positive to negative

7) If $g$ is a differentiable function such that $g(x) < 0$ for all real numbers $x$ and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

rel min of f(x)

$-$ $+$ $-$

$-2$ $2$

rel max on f(x)

(A) $f$ has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) $f$ has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) $f$ has relative minima at $x = -2$ and at $x = 2$.
(D) $f$ has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if $f$ has any relative extrema.

$f'(x) = (x^2-4) \cdot g(x)$ always negative

$f'(-3) =$ NEG
$f'(0) =$ POS
$f'(3) =$ NEG

critical points
$f'(x) = (x^2-4) \cdot g(x)$
$0 = (x^2-4) \cdot g(x)$
$x^2 - 4 = 0$ $g(x) = 0$ Never equals zero
$x = \pm 2$ DNE

8) Let $f$ be a function defined for all real numbers $x$. If $f'(x) = \dfrac{\left|4 - x^2\right|}{x - 2}$, then $f$ is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$

$-$ $-$ $+$

$-2$ $2$

$4 - x^2 = 0$
$x = \pm 2$

$x - 2 = 0$
$x = 2$

$f'(-3) =$ NEG
$f'(0) =$ NEG
$f'(3) =$ POS