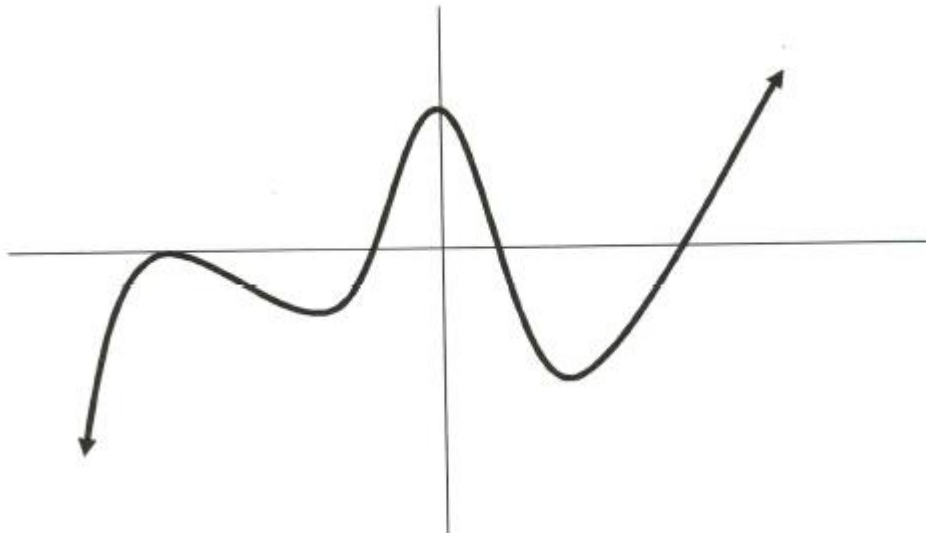


Increasing and Decreasing Functions & The First Derivative Test

Draw 10 different tangent lines to the graph below:



What is true about the slopes of a graph when the function increases (y -values increase as x -values increase left to right)?

What is true about the slopes of a graph when the function decreases (y -values decrease as x -values increase left to right)?

Tangent line slopes are **POSITIVE** on intervals where a graph increases and **NEGATIVE** on intervals where a graph decreases.

Since a graph can only change from increasing to decreasing(or vice versa) at a critical point, Calculus can be used for find intervals of increase/decrease and ordered pairs for maximums, minimums and plateaus.

THEOREM 3.5 Test for Increasing and Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b) .

1. If $f'(x) > 0$ for all x in (a, b) , then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b) , then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b) , then f is constant on $[a, b]$.

Using the First Derivative Test to find intervals of increase/decrease and x - values for relative maximums/minimums and plateaus.

- 1) Find critical numbers and place them on a number line.
- 2) Pick any value in between these critical values and evaluate them in the derivative to determine if the value is on an increasing(positive value) or decreasing(negative value) interval and mark with a +/- above the number line. List CLOSED intervals of increase and decrease.
- 3) If there is a sign change over a critical point, then POSSIBLY a min./max. occurs there. If no sign change occurs over a critical point then POSSIBLY a plateau occurs. Remember, **neither** of these could be true if the critical point represents the x -value of a vertical *asymptote*.

EX) Give an example of function with critical value of $x = 1$ which changes from increasing to decreasing at this value but **IS NOT** the x -value of a relative maximum.

Maximums/Minimums/Plateaus

When a graph changes from **increasing to decreasing** over a DEFINED critical point then a **MAXIMUM** occurs at that x -value!

When a graph changes from **decreasing to increasing** over a DEFINED critical point then a **MINIMUM** occurs at that x -value!

When a graph **DOES NOT** change from **dec. to inc. or vice-versa** over a DEFINED critical point then a **PLATEAU** occurs at that x -value!

EX For each function, determine(without a calculator) intervals of inc/dec and identify any ordered pairs of relative maximums/minumums or plateaus.

A) $f(x) = x^3 - \frac{3}{2}x^2$

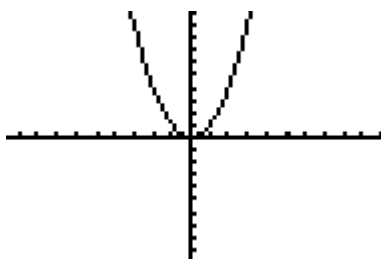
B) $f(x) = x^3 + 6x^2 + 12x + 8$

C) $f(x) = (x^2 - 9)^{2/3}$

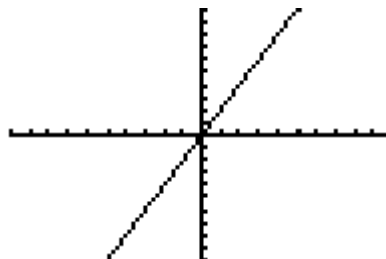
READING A DERIVATIVE GRAPH TO DETERMINE TRAITS OF ORIGINAL FUNCTION

Compare the graphs of a function and its derivative below.

$$f(x) = x^2$$



$$f'(x) = 2x$$



Notice when reading the graph of the derivative, the y-values represent the **slope** of the graph of the original function at the same x-value.

When reading a derivative graph($f'(x)$):

x-intercepts represent x-values where horizontal tangents occur on original function

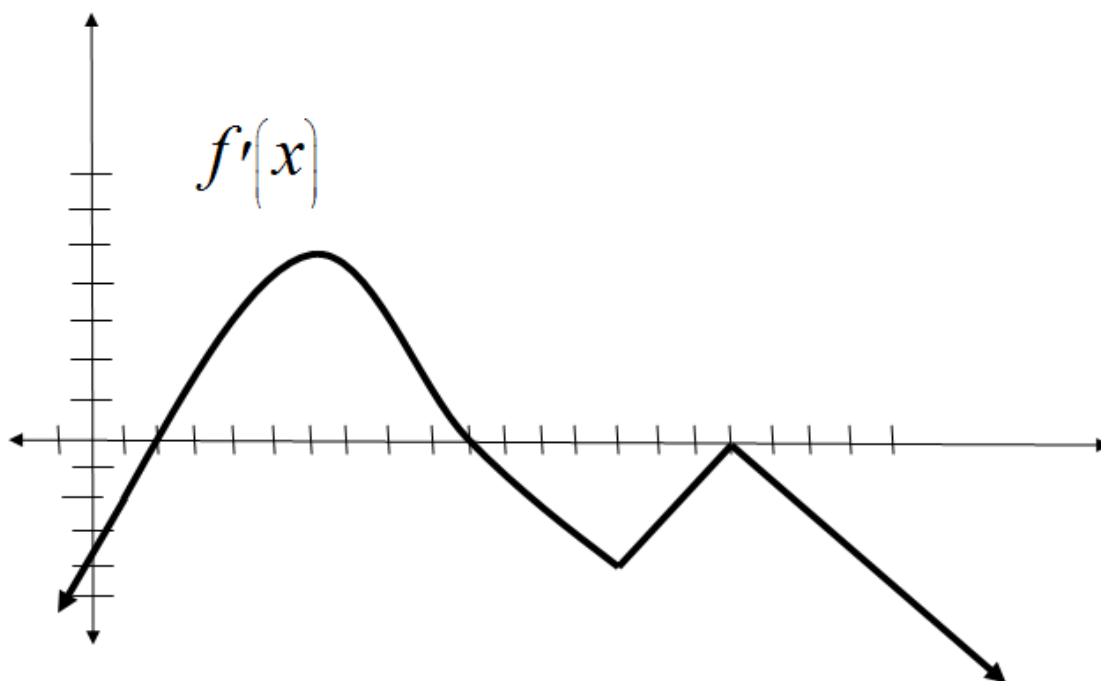
AND

intervals where there are positive y-values(above the x-axis) *on the derivative* represent intervals of increase on the original function

AND

intervals where there are negative y-values(below the x-axis) *on the derivative* represent intervals of decrease on the original function

EX) Use the graph of the **derivative** below to list intervals of increase/decrease **on $f(x)$** and identify any x -values of minimums, maximums and plateaus.

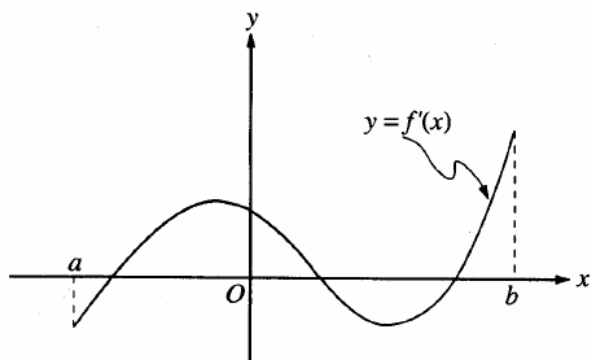


Using the Calculator to Determine Intervals of INC/DEC

If $f'(x) = \sin x \bullet \ln(x^2 - 1)$, determine the interval(s) of increase/decrease of $f(x)$ over $[2, 10]$. Also, determine the x -values of any relative extrema on $f(x)$.

AP EXAMPLES USING GRAPHS

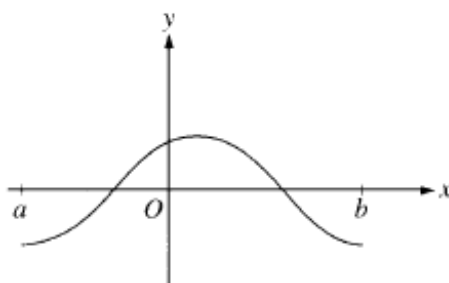
EX1)



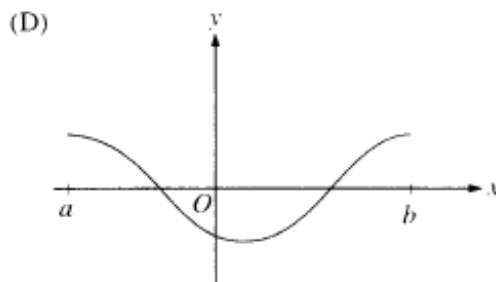
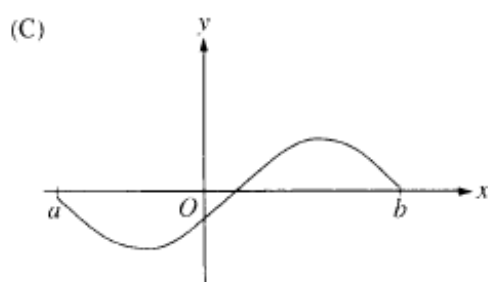
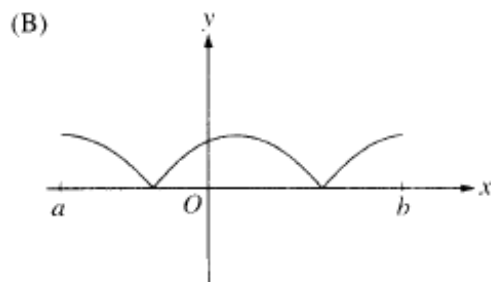
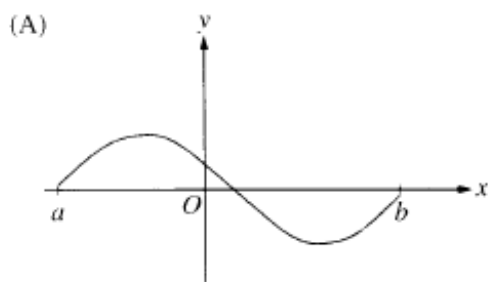
The graph of f' , the derivative of f , is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a, b) ?

- (A) One relative maximum and two relative minima
- (B) Two relative maxima and one relative minimum
- (C) Three relative maxima and one relative minimum
- (D) One relative maximum and three relative minima
- (E) Three relative maxima and two relative minima

EX2)



The graph of f is shown in the figure above. Which of the following could be the graph of the derivative of f ?



When does *Velocity & Speed* INC/DEC?

Given an object thrown straight into the air with velocity of 88 ft/sec from 25 feet can be represented by $H(t) = -16t^2 + 88t + 25$.

EX) Find the time when the velocity is zero and explain what this means in context of the moving object?

EX) What is the velocity at $t = 1$ second? Is velocity increasing or decreasing at this time? Hint: see if $V'(1)$ is POS or NEG since the sign of the derivative of velocity determines if actual velocity is increasing or decreasing.

EX) What is the velocity at $t = 4$ seconds? Is velocity increasing or decreasing at this time? Hint: see if $V'(4)$ is POS or NEG!

Since **acceleration** is the derivative of velocity($V'(t)$), it can be used to find when the velocity increases or decreases(**positive** acceleration means velocity is **increasing** and **negative** acceleration means velocity **decreasing**)

EX) What is the **speed** at $t = 1$ and $t = 4$ seconds? Is the **speed** increasing or decreasing at each of these times? Note: You cannot use the sign of $V'(t)$ alone as velocity is not always the same as speed.

So.....**When velocity and acceleration have the same sign, the SPEED is INCREASING(getting faster) and when velocity and acceleration have different signs, the SPEED is DECREASING(slowing down)!**