Find the interval(s) on which the function is concave up/concave down and list any inflection points on the graph of the function.

1a) $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x + 2)$

$f''(x) = 0$ when $x = 0, -2$

Concave Up: $(-\infty,-2]\cup[0,\infty)$

Concave Down: $[-2,0]$

Points of Inflection: $(-2,-8)\,\&\,(0,0)$

1b) $f(x) = x^3 - 6x^2 + 12x$

1c) $f(x) = x(x - 4)^3$

1d) $f(x) = \sin x + \cos x, \quad [0, 2\pi]$

1e) $f(x) = \dfrac{4}{x^2 + 1}$

2a) Let g be a twice differentiable function, and let $g(-6) = -1$, $g'(-6) = 0$, and $g''(-6) = -3$.

What occurs in the graph of g at the point $(-6, -1)$?

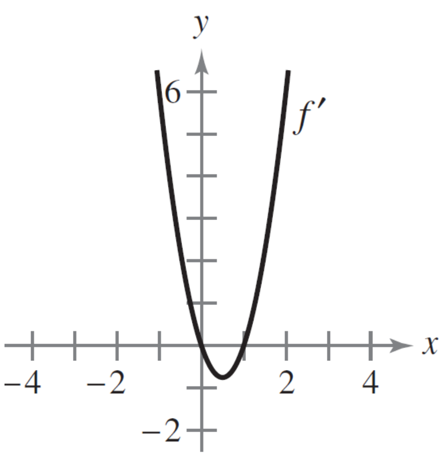
Since the slope at (-6, -1) is zero and occurs on a concave down portion of the graph(because $g''(-6)$ is negative), it must be a **relative maximum**.

2b) Let h be a twice differentiable function, and let $h(5) = 1$, $h'(5) = 0$, and $h''(5) = 2$.

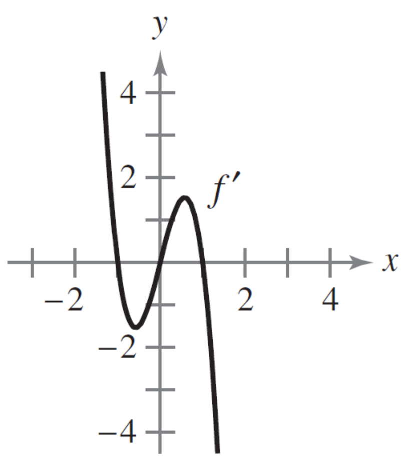
What occurs in the graph of h at the point $(5, 1)$?

Use the graph of $f'(x)$ to (a) identify the interval(s) on which $f(x)$ is concave down or concave up, and (b) estimate the value(s) of x at which $f(x)$ has an inflection point.

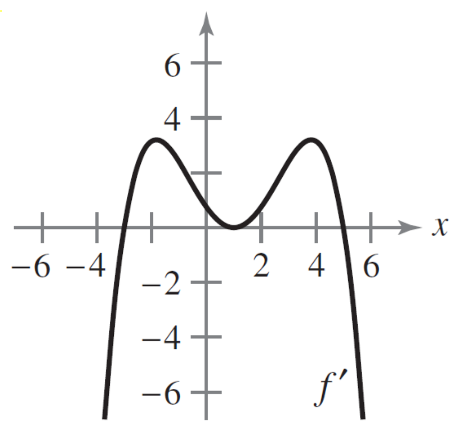
3a)



3b)



3c)



Concave Up: $\left[\frac{1}{2}, \infty\right)$

Concave Down: $\left(-\infty, \frac{1}{2}\right]$

Inflection point at $x = \frac{1}{2}$

Sketch the graph of a function having the given characteristics.

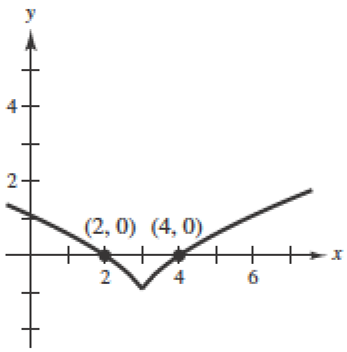
4a) $f(2) = f(4) = 0$

$f'(x) < 0$ if $x < 3$

$f'(3)$ does not exist.

$f'(x) > 0$ if $x > 3$

$f''(x) < 0, x \neq 3$



4b) $f(2) = f(4) = 0$

$f'(x) > 0$ if $x < 3$

$f'(3)$ does not exist

$f'(x) < 0$ if $x > 3$

$f''(x) > 0, x \neq 3$

Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

5a) The graph of every cubic polynomial has precisely one point of inflection.

5b) The graph of $f(x) = 1/x$ is concave downward for $x < 0$ and concave upward for $x > 0$, and thus it has a point of inflection at $x = 0$.

5c) If $f'(c) > 0$, then f is concave upward at $x = c$.

5d) If $f''(2) = 0$, then the graph of f must have a point of inflection at $x = 2$.

6) AP MULTIPLE CHOICE EXAMPLES

1) The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) $(0,0)$ only (B) $(3,162)$ only (C) $(4,256)$ only

(D) $(0,0)$ and $(3,162)$ (E) $(0,0)$ and $(4,256)$

2) Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$

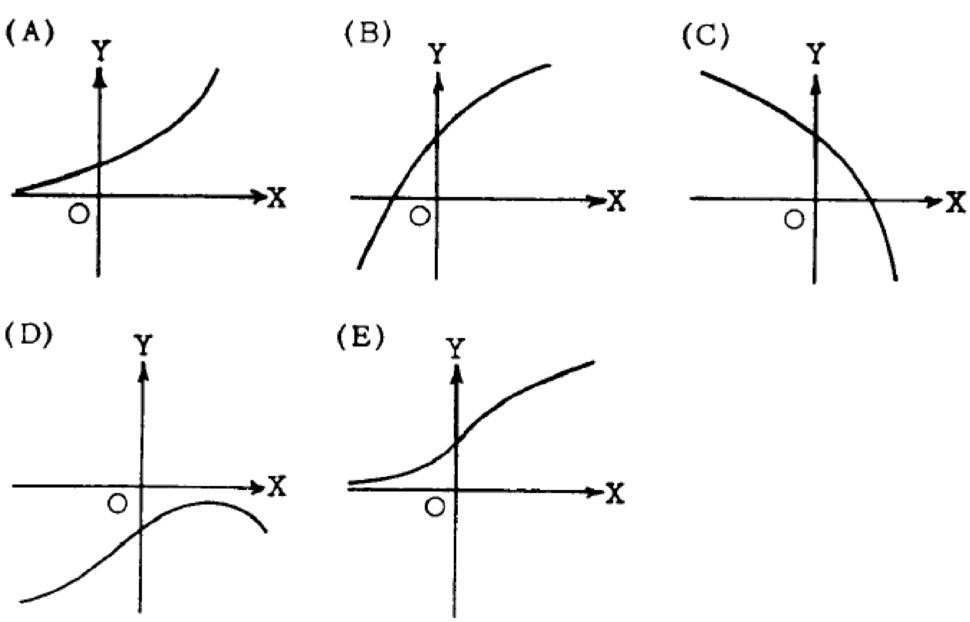
(B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

(C) $-2 < x < 0$ or $x > 2$

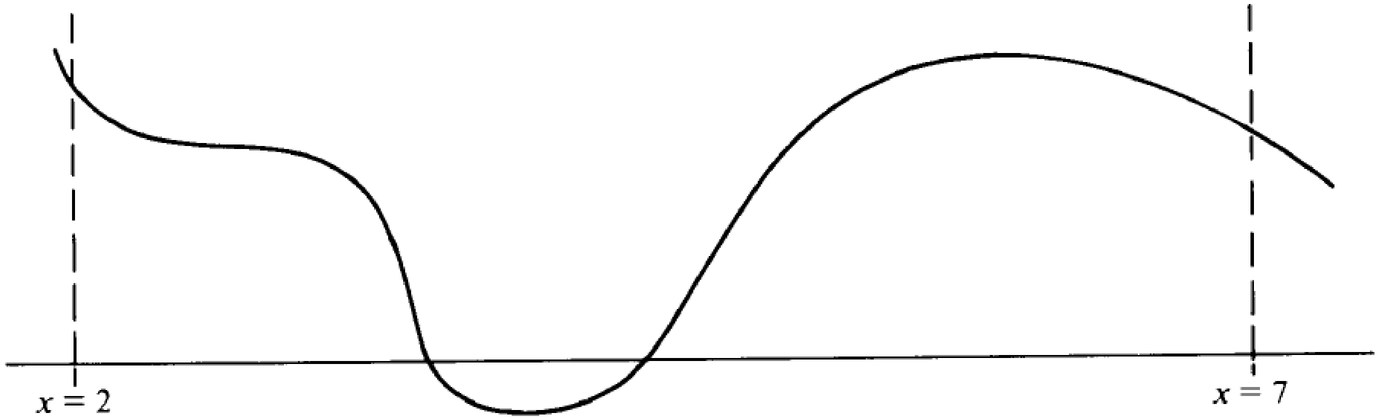
(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

3) If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?



4)

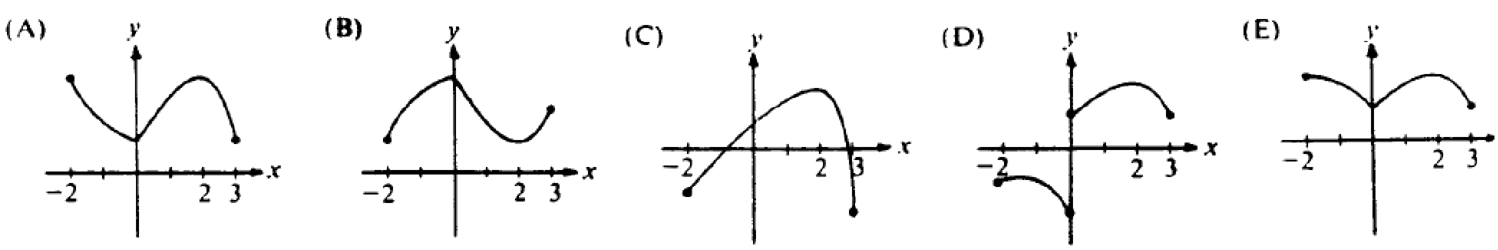


The graph of $y = f(x)$ on the closed interval $[2, 7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two (C) Three (D) Four (E) Five

5)

Let f be a function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?



6) For all x in the closed interval [2, 5], the function f has $f'(x) > 0$ and $f''(x) < 0$. Which of the following could be the table of values for f?

(A)

x	$f(x)$
2	7
3	11
4	14
5	16

(B)

x	$f(x)$
2	7
3	9
4	12
5	16

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

7) Let f be a twice differentiable function with $f'(x) > 0$ and $f''(x) > 0$ for all real numbers x, such that $f(4) = 12$ **and** $f(5) = 18$. Of the following, what is a possible value for $f(6)$?

A) 27 **B) 24** **C) 21** **D) 18** **E) 15**