Find the interval(s) on which the function is concave up/concave down and list any inflection points on the graph of the function.

1a) $f(x) = \frac{1}{2}x^4 + 2x^3$

$f'(x) = 2x^3 + 6x^2$

$f''(x) = 6x^2 + 12x = 6x(x+2)$

$f''(x) = 0$ when $x = 0, -2$

Concave Up: $(-\infty,-2]\cup[0,\infty)$

Concave Down: $[-2,0]$

Points of Inflection: $(-2,-8)$ & $(0,0)$

1b) $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12$

$f''(x) = 6x - 12$

$0 = 6x - 12$

$2 = x$

$-$ $+$

2

CC up $[2,\infty)$ CC DOWN $(-\infty,2]$

inf point when $x=2$

$f(2) = 8$

inf. point $(2,8)$

1c) $f(x) = x(x-4)^3$

$f'(x) = x\cdot 3(x-4)^2 + (x-4)^3\cdot 1$

$f'(x) = (x-4)^2\cdot(3x + (x-4))$

$f'(x) = (x-4)^2(4x-4)$

$f''(x) = (x-4)^2\cdot 4 + (4x-4)\cdot 2(x-4)^1\cdot 1$

$f''(x) = (x-4)^1(4(x-4) + 2(4x-4))$

$f''(x) = (x-4)(4x-16+8x-8)$

$f''(x) = (x-4)\cdot(12x-24)$

$0 = (x-4)\cdot(12x-24)$

$x-4=0$ $12x-24=0$

$x=4$ $x=2$

$+$ $-$ $+$

2 4

CC up $(-\infty,2]\cup[4,\infty)$

CC DOWN $[2,4]$

inf pts at $x = 2$ & 4

$f(2) = -16$ $f(4) = 0$

Inf points $(2,-16)$ & $(4,0)$

1d) $f(x) = \sin x + \cos x,\ [0, 2\pi]$

$f'(x) = \cos x + (-\sin x)$

$f''(x) = -\sin x + (-\cos x)$

$0 = -\sin x - \cos x$

$\frac{\cos x}{\cos x} = -\frac{\sin x}{\cos x}$

$1 = -\tan x$

$-1 = \tan x$

$x = \frac{3\pi}{4}$ & $\frac{7\pi}{4}$

$-$ $+$ $-$

0 $\frac{3\pi}{4}$ $\frac{7\pi}{4}$ 2π

CC up $[\frac{3\pi}{4},\frac{7\pi}{4}]$

CC DOWN $[0,\frac{3\pi}{4}]\cup[\frac{7\pi}{4},2\pi]$

inf. pts at $x = \frac{3\pi}{4}$ & $\frac{7\pi}{4}$

$f(\frac{3\pi}{4}) = 0$ $f(\frac{7\pi}{4}) = 0$

inf. points $(\frac{3\pi}{4},0)$ & $(\frac{7\pi}{4},0)$

1e) $f(x) = \dfrac{4}{x^2+1}$

$f(x) = 4(x^2+1)^{-1}$

$f'(x) = -4(x^2+1)^{-2}\cdot 2x$

$f'(x) = \dfrac{-8x}{(x^2+1)^2}$

$f''(x) = \dfrac{(x^2+1)^2\cdot(-8) - (-8x)2(x^2+1)^1\cdot 2x}{(x^2+1)^4}$

$f''(x) = \dfrac{-8(x^2+1)^2 + 32x^2(x^2+1)}{(x^2+1)^4}$

$f''(x) = \dfrac{\cancel{(x^2+1)}(-8(x^2+1)+32x^2)}{(x^2+1)^{\cancel{4}3}}$

$f''(x) = \dfrac{-8x^2+32x^2-8}{(x^2+1)^3}$

$0 = \dfrac{24x^2-8}{(x^2+1)^3}$

$0 = \dfrac{24x^2-8}{(x^2+1)^3}$

$0 = 24x^2-8$

$\frac{1}{3} = x^2$

$\pm\frac{1}{\sqrt{3}} = x$

$+$ $-$ $+$

$-\frac{1}{\sqrt{3}}$ $\frac{1}{\sqrt{3}}$

CCUP $(-\infty,-1/\sqrt{3}]\cup[1/\sqrt{3},\infty)$

CC DOWN $[-1/\sqrt{3},1/\sqrt{3}]$

Inf points at $x = \pm 1/\sqrt{3}$

$f(-1/\sqrt{3}) = 3$ $f(1/\sqrt{3}) = 3$

inf points $(-1/\sqrt{3},3)$ & $(1/\sqrt{3},3)$

2a) Let g be a twice differentiable function, and let $g(-6) = -1$, $g'(-6) = 0$, and $g''(-6) = -3$.

What occurs in the graph of g at the point $(-6, -1)$?

Since the slope at (-6, -1) is zero and occurs on a concave down portion of the graph(because $g''(-6)$ is negative), it must be a **relative maximum**.

2b) Let h be a twice differentiable function, and let $h(5) = 1$, $h'(5) = 0$, and $h''(5) = 2$.

What occurs in the graph of h at the point $(5, 1)$?

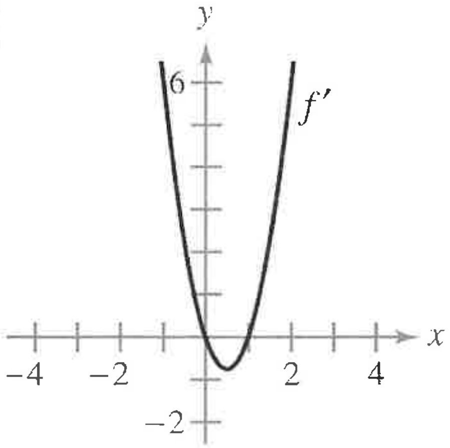
slope is zero at $x=5$ (so at the point (5,1))

$h''(5) > 0$ so graph is CC up at the point (5,1)

So... (5,1) must be a relative MINIMUM.

Use the graph of $f'(x)$ to (a) identify the interval(s) on which $f(x)$ is concave down or concave up, and (b) estimate the value(s) of x at which $f(x)$ has an inflection point.

3a)

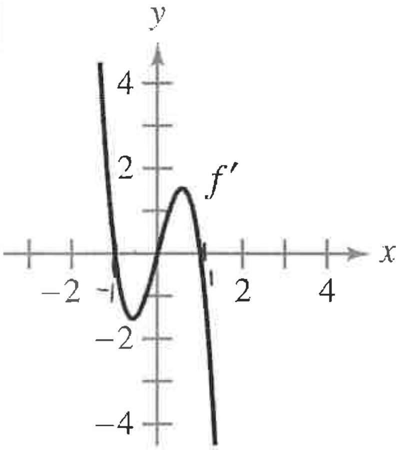


Concave Up: $\left[\frac{1}{2}, \infty\right)$

Concave Down: $\left(-\infty, \frac{1}{2}\right]$

Inflection point at $x = \frac{1}{2}$

3b)

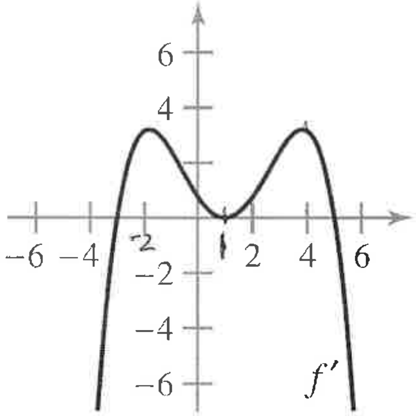


CC UP $[-1/2, 1/2]$

CC DOWN $(-\infty, -1/2] \cup [1/2, \infty)$

Inf points at $x = -1/2$ & $1/2$

3c)



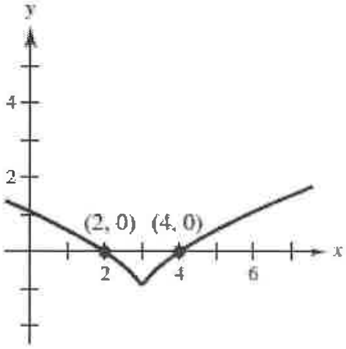
CC UP $(-\infty, -2] \cup [1, 4]$

CC DOWN $[-2, 1] \cup [4, \infty)$

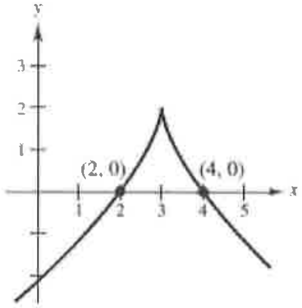
Inf. points at $x = -2, 1$ & 4

Sketch the graph of a function having the given characteristics.

4a) $f(2) = f(4) = 0$
$f'(x) < 0$ if $x < 3$
$f'(3)$ does not exist
$f'(x) > 0$ if $x > 3$
$f''(x) < 0, x \neq 3$



4b) $f(2) = f(4) = 0$
$f'(x) > 0$ if $x < 3$
$f'(3)$ does not exist
$f'(x) < 0$ if $x > 3$
$f''(x) > 0, x \neq 3$



Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

5a) The graph of every cubic polynomial has precisely one point of inflection.

TRUE: Second derivative WILL be linear yielding ONLY one solution when set equal to zero.

5b) The graph of $f(x) = 1/x$ is concave downward for $x < 0$ and concave upward for $x > 0$, and thus it has a point of inflection at $x = 0$.

FALSE: There is an asymptote at $x = 0$. Concavity can also change where 2nd derivative is undefined!

5c) If $f'(c) > 0$, then f is concave upward at $x = c$.

FALSE: Function is INCREASING at $x = c$. But, it does not necessarily mean the SLOPES are increasing(cc up) there! This WOULD be true if $f''(c) > 0$

5d) If $f''(2) = 0$, then the graph of f must have a point of inflection at $x = 2$.

FALSE: When the first derivative graph has a plateau at $x = 2$, $f''(2) = 0$, yet the graph of the derivative would continue to increase or decrease, implying concavity is not changing there.

6) AP MULTIPLE CHOICE EXAMPLES

1) The graph of $y = 5x^4 - x^5$ has a point of inflection at

(A) (0,0) only [✓ (B) (3,162) only] (C) (4,256) only

(D) (0,0) and (3,162) (E) (0,0) and (4,256)

$y' = 20x^3 - 5x^4$

$y'' = 60x^2 - 20x^3$

$20x^2 = 0$ $3 - x = 0$ $0 = 20x^2(3 - x)$

$x^2 = 0$ $3 = x$

$x = \pm\sqrt{0}$ $x = 0$ or $x = 3$

+ + −

0 3

$y(3) = 162$

2) Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

(A) $x > 0$

[✓ (B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$]

(C) $-2 < x < 0$ or $x > 2$

(D) $x > \sqrt{2}$

(E) $-2 < x < 2$

$f'(x) = 15x^4 - 60x^2$

$f''(x) = 60x^3 - 120x$

$0 = 60x \cdot (x^2 - 2)$

$60x = 0$ $x^2 - 2 = 0$

$x = 0$ $x^2 = 2$

$x = \pm\sqrt{2}$

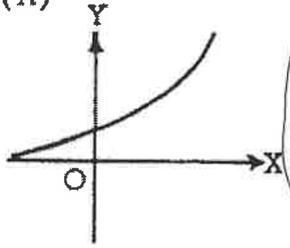
− + − +

$-\sqrt{2}$ 0 $\sqrt{2}$

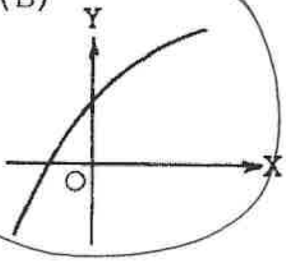
INC CC DOWN

3) If y is a function of x such that $y' > 0$ for all x and $y'' < 0$ for all x, which of the following could be part of the graph of $y = f(x)$?

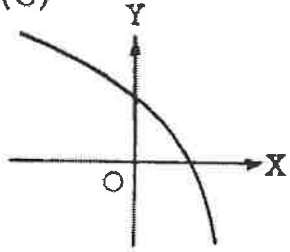
(A)



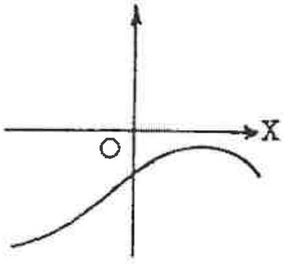
(B)



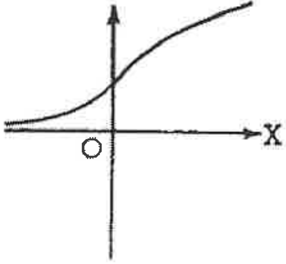
(C)



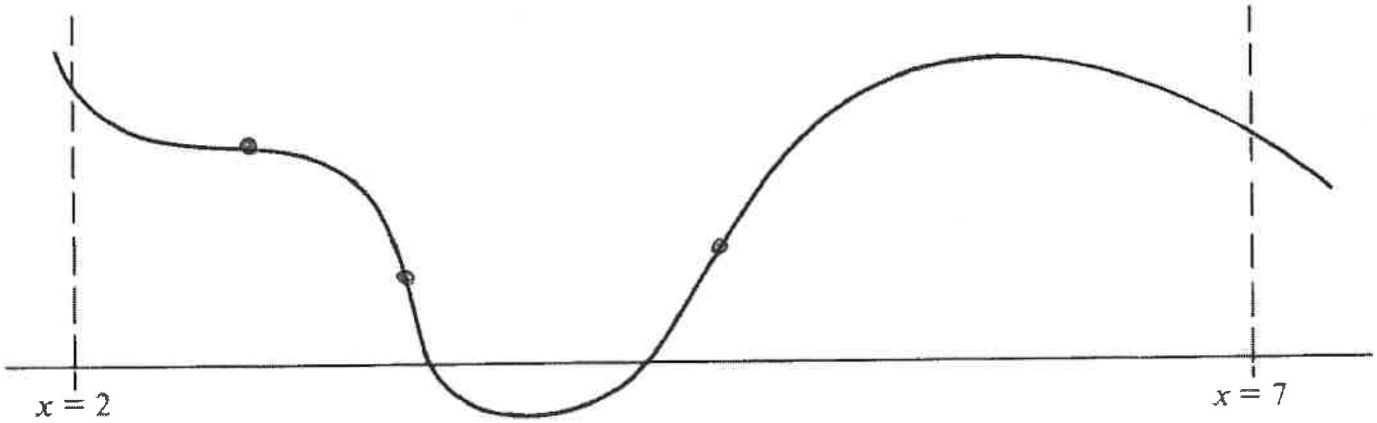
(D)



(E)



4)



The graph of $y = f(x)$ on the closed interval $[2,7]$ is shown above. How many points of inflection does this graph have on this interval?

(A) One (B) Two [✓ (C) Three] (D) Four (E) Five

sharp turn

at $x=0$

5)

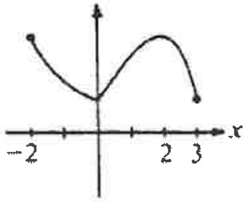
Let f be a function that is continuous on the closed interval $[-2,3]$ such that $f'(0)$ does not exist, $f'(2)=0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?

$x=2$ flat

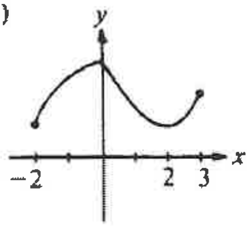
horiz tang.

CC DOWN

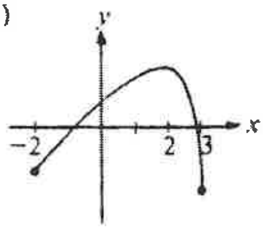
(A)



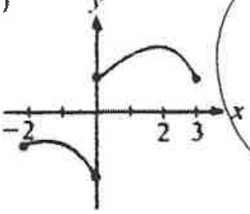
(B)



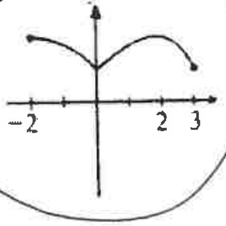
(C)



(D)



[✓ (E)]



f(x) increases (as x gets bigger so does "y")

f(x) slopes decrease (cc down)

6) For all x in the closed interval [2, 5], the function f has $f'(x)>0$ and $f''(x)<0$. Which of the following could be the table of values for f?

[✓ (A)]

x	$f(x)$
2	7
3	11
4	14
5	16

up 4 over 1

up 3 over 1

up 2 over 1

(B)

x	$f(x)$
2	7
3	9
4	12
5	16

up 2 over 1

up 3 over 1

up 4 over 1

(C)

x	$f(x)$
2	16
3	12
4	9
5	7

(D)

x	$f(x)$
2	16
3	14
4	11
5	7

(E)

x	$f(x)$
2	16
3	13
4	10
5	7

C, D, E decrease so NOT THE ANSWER

only A increases while its slopes DECREASE

f(x) increases

f(x) slopes increase (cc up)

7) Let f be a twice differentiable function with $f'(x)>0$ and $f''(x)>0$ for all real numbers x, such that $f(4)=12$ **and** $f(5)=18$. Of the following, what is a possible value for $f(6)$?

[✓ A) 27] B) 24 C) 21 ~~D) 18~~ ~~E) 15~~

If f(x) increases, it must be bigger than 18

the slope from 4 to 5 $= \frac{18-12}{5-4} = \frac{6}{1}$

the slope from 5 to 6 must be bigger than 6 because slopes are increasing

If f(6) = 27 slope from 5 to 6 $= \frac{27-18}{6-5} = \boxed{9}$

bigger than "6"