**Find the interval(s) on which the function is increasing or decreasing, and identify all relative extrema.**

1a)

$$f(x) = x^2 - 4x$$

(a) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

Critical number: $x = 2$

(b)

| Test intervals: | $-\infty < x < 2$ | $2 < x < \infty$ |
|---|---|---|
| Sign of $f'$: | $f' < 0$ | $f' > 0$ |
| Conclusion: | Decreasing | Increasing |

Decreasing on: $(-\infty, 2]$

Increasing on: $[2, \infty)$

(c) Relative minimum: $(2, -4)$

1b) $f(x) = \dfrac{x^5 - 5x}{5}$

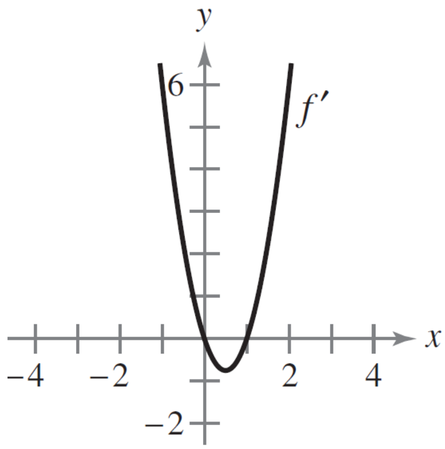
1c) $f(x) = (x - 1)^2(x + 3)$

1d) $f(x) = \dfrac{x^2}{x^2 - 9}$

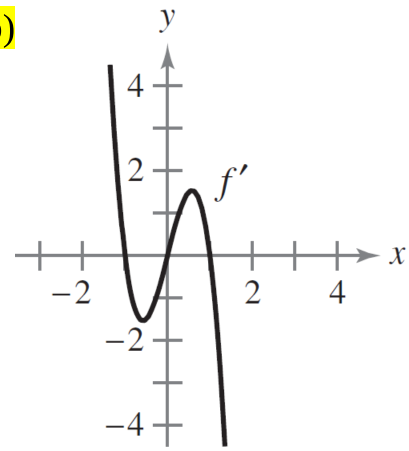
1e) $f(x) = \begin{cases} 4 - x^2, & x \le 0 \\ -2x, & x > 0 \end{cases}$

**Use the graph of $f'(x)$ to (a) identify the interval(s) on which $f(x)$ is increasing or decreasing, and (b) estimate the value(s) of *x* at which $f(x)$ has a relative maximum or minimum.**

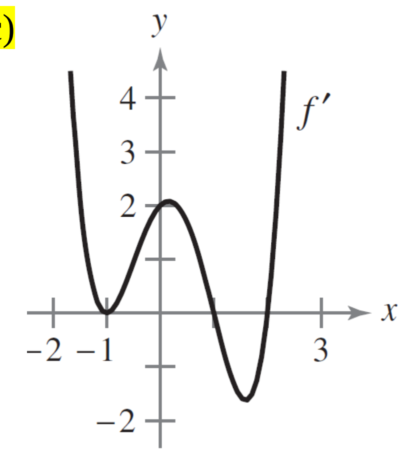
2a)




2b)




2c)




(a) Intervals of Increase $(-\infty,\ 0] \cup [1,\ \infty)$

Intervals of Decrease $[0,\ 1]$

(b) Relative maximum: $x = 0$

Relative minimum: $x = 1$

3) AP MULTIPLE CHOICE EXAMPLES

1) If $f(x) = x + \frac{1}{x}$, then the set of values for which $f$ increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$

(D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

2) A point moves on the $x$-axis in such a way that its velocity at time $t$ $(t > 0)$ is given by $v = \frac{\ln t}{t}$. At what value of $t$ does $v$ attain its maximum?

(A) 1 (B) $e^{\frac{1}{2}}$ (C) $e$ (D) $e^{\frac{3}{2}}$

(E) There is no maximum value for $v$.

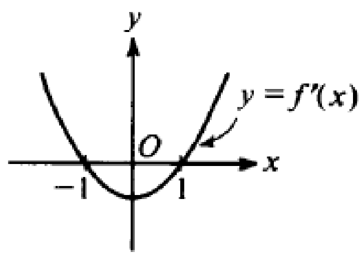
3) For what value of $k$ will $x+\frac{k}{x}$ have a relative maximum at $x=-2$?

(A) $-4$ (B) $-2$ (C) $2$ (D) $4$ (E) None of these

4) At $x=0$, which of the following is true of the function $f$ defined by $f(x)=x^2+e^{-2x}$ ?

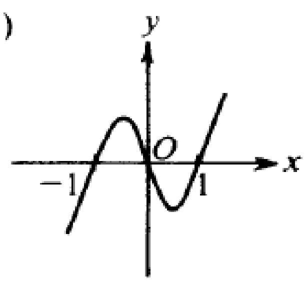
(A) $f$ is increasing.
(B) $f$ is decreasing.
(C) $f$ is discontinuous.
(D) $f$ has a relative minimum.
(E) $f$ has a relative maximum.

5)

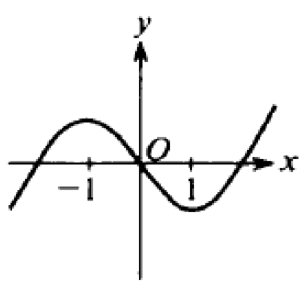



The graph of the derivative of $f$ is shown in the figure above. Which of the following could be the graph of $f$?

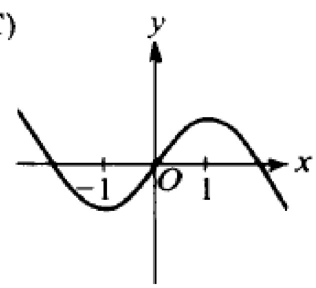
(A)



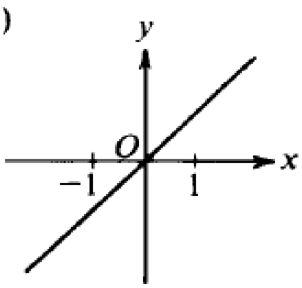

(B)



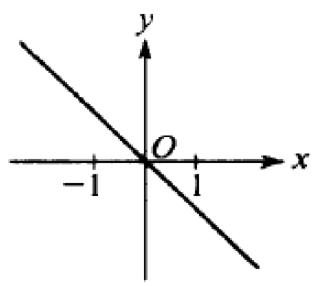

(C)



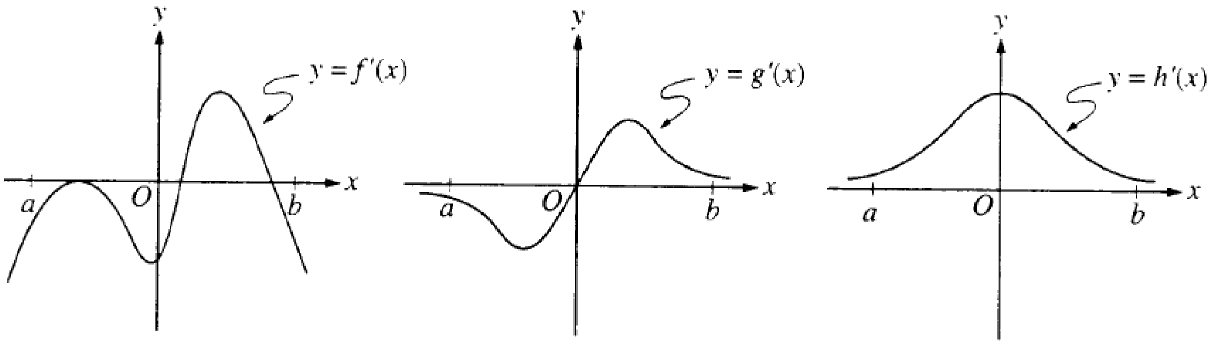

(D)




(E)



6)




The graphs of the derivatives of the functions $f$, $g$, and $h$ are shown above. Which of the functions $f$, $g$, or $h$ have a relative maximum on the open interval $a < x < b$?

(A) $f$ only
(B) $g$ only
(C) $h$ only
(D) $f$ and $g$ only
(E) $f$, $g$, and $h$

7) If $g$ is a differentiable function such that $g(x) < 0$ for all real numbers $x$ and if $f'(x) = \left(x^2 - 4\right)g(x)$, which of the following is true?

(A) $f$ has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) $f$ has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) $f$ has relative minima at $x = -2$ and at $x = 2$.
(D) $f$ has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if $f$ has any relative extrema.

8) Let $f$ be a function defined for all real numbers $x$. If $f'(x) = \dfrac{\left|4 - x^2\right|}{x - 2}$, then $f$ is decreasing on the interval

(A) $(-\infty, 2)$ (B) $(-\infty, \infty)$ (C) $(-2, 4)$ (D) $(-2, \infty)$ (E) $(2, \infty)$