Given $f \circ g(x) = x$, use implicit differentiation to find *derivative* of $g(x)$ at the given point.

point on inverse of $f(x)$

1a) $f(x) = x^3 - 7x^2 + 2$ and $g(-4) = 1$

$$x = y^3 - 7y^2 + 2$$

$$1 = 3y^2\frac{dy}{dx} - 14y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3y^2 - 14y}$$

At $(-4, 1)$, $\frac{dy}{dx} = \frac{1}{3 - 14} = \frac{-1}{11}$

1b) $f(x) = 2\ln(x^2 - 3)$ and $g(0) = 2$

INVERSE $\Rightarrow$ $x = 2\ln(y^2 - 3)$

$$1 = 2 \cdot \frac{1}{y^2 - 3} \cdot 2y\frac{dy}{dx}$$

$$1 = \frac{4y}{y^2-3}\frac{dy}{dx}$$

deriv. of inverse $\Rightarrow$ $\frac{y^2-3}{4y} = \frac{dy}{dx}$

$$\frac{2^2-3}{4(2)} = \frac{dy}{dx}(0,2)$$

$$\boxed{\frac{1}{8}} = g'(0)$$

2) AP MULTIPLE CHOICE EXAMPLES

1) If the function f is defined by $f(x) = x^5 - 1$, then f^{-1}, the inverse function of f, is defined by $f^{-1}(x) =$

$y = x^5 - 1$

INV $\Rightarrow$ $x = y^5 - 1$

$x + 1 = y^5$

$\sqrt[5]{x+1} = y$

(A) $\frac{1}{\sqrt[5]{x}+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$

(D) $\sqrt[5]{x} - 1$ (E) $\sqrt[5]{x+1}$

2) Let f and g be functions that are differentiable everywhere. If $\boxed{g \text{ is the inverse function of } f}$ and if $g(-2) = 5$ and $f'(5) = -\frac{1}{2}$, then $g'(-2) =$ slope of $g(x)$ at $(-2, 5)$

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{5}$ (D) $-\frac{1}{5}$ (E) -2

$g(x)$ contains $(-2, 5)$

$f(x)$ contains $(5, -2)$

if $f'(5) = \frac{-1}{2}$ then $g'(-2) = -2$

3) The function f is defined by $f(x) = x^3 + 4x + 2$. If g is the inverse function of f and $g(2) = 0$, what is the value of $g'(2)$?

(A) $-\frac{1}{16}$ (B) $-\frac{4}{81}$ (C) $\frac{1}{4}$ (D) 4

$g(x)$ contains $(2,0)$ ⟵ $g'(2)$ is slope at this point

$f(x)$ contains $(0,2)$ ⟵ slope of this point is 4

$f'(x) = 3x^2+4$

$f'(0) = 4$

4) If $f(x) = \sin x$, then $(f^{-1})'(\frac{\sqrt{3}}{2}) =$

(A) $\frac{1}{2}$ (B) $\frac{2\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) 2

$f^{-1}(x) = \arcsin x$

$[f^{-1}(x)]' = \frac{1}{\sqrt{1-x^2}}$

$[f^{-1}(\frac{\sqrt{3}}{2})]' = \frac{1}{\sqrt{1-(\frac{\sqrt{3}}{2})^2}}$

$= \frac{1}{\sqrt{1-\frac{3}{4}}}$

$= \frac{1}{\sqrt{\frac{1}{4}}}$

$= \frac{1}{\frac{1}{2}}$

$= 2$

OR

$f^{-1}(x)$ contains $(\frac{\sqrt{3}}{2}, ?)$

$f(x)$ contains $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$ ⟵ must be $\frac{\pi}{3}$ since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$f'(x) = \cos x$

$f'(\frac{\pi}{3}) = \frac{1}{2}$

so.. $[f^{-1}(\frac{\sqrt{3}}{2})]' = 2$