Keystone Algebra I Review

(Module 1 Answers)

Compare an Order Real Number

Inequality symbols

< means less than

> means greater than

EX) Order the values from least to greatest using inequality symbols

$$\pi/3,\ \sqrt{7},\ 2.91,\ -\sqrt{5},\ -8$$

Use a calculator to convert to decimals. Remember, the larger the actual value of a negative number is the SMALLER it is!

Since $\frac{\pi}{3} \approx 1.047 \quad \sqrt{7} \approx 2.646 \quad -\sqrt{5} \approx -2.236$

$$-8 < -\sqrt{5} < \frac{\pi}{3} < \sqrt{7} < 2.91$$

EX) Is the following inequality true or false when x = 4?

$$3x^2 + 2 \le x^3 - 9$$

1) Plug the "4" in for the x

2) Use order of operations to simplify and answer the question.

$$3 \bullet 4^2 + 2 \le 4^3 - 9$$
$$3 \bullet 16 + 2 \le 64 - 9$$
$$48 + 2 \le 64 - 9$$
$$50 \le 55$$

TRUE

Simplify Square Roots

Square root must be as SMALL as possible meaning no perfect squares can be factored out.

Square Roots that give whole number answers:

$\sqrt{4}=2$ $\sqrt{9}=3$ $\sqrt{16}=4$ $\sqrt{25}=5$ $\sqrt{36}=6$ $\sqrt{49}=7$ $\sqrt{64}=8$ $\sqrt{81}=9$

$\sqrt{100}=10$ $\sqrt{121}=11$ $\sqrt{144}=12$ $\sqrt{169}=13$....................

EX) Simplify

$$\sqrt{80}=\sqrt{4}\bullet\sqrt{20}$$
$$=2\bullet\sqrt{20}$$
$$=2\bullet\sqrt{4}\bullet\sqrt{5}$$
$$=2\bullet2\bullet\sqrt{5}$$
$$=4\bullet\sqrt{5}$$

EX) $\sqrt{54}=\sqrt{9}\bullet\sqrt{6}=3\bullet\sqrt{6}$

Note: You may **multiply** or **divide** square roots BEFORE or AFTER simplifying

EX)

$$\sqrt{64\bullet18}$$
$$=\sqrt{64}\bullet\sqrt{18}$$
$$=8\bullet\sqrt{18}$$
$$=8\bullet\sqrt{9}\bullet\sqrt{2}$$
$$=8\bullet3\bullet\sqrt{2}$$
$$=24\bullet\sqrt{2}$$

EX) $\dfrac{\sqrt{500}}{\sqrt{10}}=\sqrt{50}=\sqrt{25}\bullet\sqrt{10}=5\bullet\sqrt{10}$

Note: You may NOT add or subtract square roots but only combine LIKE roots!

EX) $\sqrt{2}+\sqrt{3}$ **cannot be combined**

EX) $4\sqrt{2}+5\sqrt{2}=9\sqrt{2}$

EX) $\sqrt{27}+2\sqrt{3}$ (simplify before deciding if you can combine)

$$=\sqrt{9}\bullet\sqrt{3}+2\sqrt{3}$$
$$=3\sqrt{3}+2\sqrt{3}$$
$$=5\sqrt{3}$$

EX) Find the smallest integer value for x that makes the radical simplify to a whole number.

$$\sqrt{27x}$$

Pick **1** for x and see if the square root works. If not try **2**. If not try **3, etc.**

$\sqrt{27\bullet1}=\sqrt{27}$ NO! Not a whole number answer

$\sqrt{27\bullet2}=\sqrt{54}$ NO! Not a whole number answer

$\sqrt{27\bullet3}=\sqrt{81}$ YES! $\sqrt{81}=9$ which IS a whole number answer

GCF & LCM

Greatest Common Factor: **Largest** expression that evenly divides into ALL expressions used (so it cannot be greater than any of the expressions)

EX) Find the **GCF** for $16x^2y^3$ and $6x^3y$

Greatest number that divides into 16 and 6 is 2

Greatest power of x that divides into x^2 and x^3 is x^2

Greatest power of y that divides into y^3 and y is y

So the answer is $2x^2y$

EX) Find the **GCF** for $12x^2y^3$ and $4x\ y$ $4xy$

Least Common Multiple: **Smallest**(Least) possible answer that ALL expressions used evenly divide INTO (so it cannot be less than any of the expressions)

EX) Find the **LCM** for $16x^2y^3$ and $6x^3y$

Smallest number that both 16 and 6 divide evenly into is 48

Smallest power of x that both x^2 and x^3 divide evenly into is x^3

Smallest power of x that both y^3 and y divide evenly into is y^3

So the answer is $48x^3y^3$

EX) Find the **LCM** for $12x^2y^3$ and $4x\ y$ $12x^2y^3$

Simplify and Evaluate Expressions

Order of Operations: Parentheses, Exponents, Multiplication & Division (from left to right) and Addition & Subtraction (from left to right)

EX) Evaluate each expression

A) $2|3x-1|+2$ when $x=-3$

$=2|3\bullet(-3)-1|+2$

$=2|-10|+2$

$=2\bullet10+2$ = 22

B) $(3x^3+1)^2$ when $x=2$

$=(3\bullet2^3+1)^2$

$=(3\bullet8+1)^2$

$=(24+1)^2$

$=(25)^2 = 625$

C) $\sqrt{2x^2+4}$ when $x=4$

$\sqrt{2\bullet4^2+4}$

$\sqrt{2\bullet16+4}$

$\sqrt{32+4}$

$\sqrt{36}= 6$

Note: Negative exponents can be simplified by evaluating them as a positive exponent in the other part (numerator or denominator) of the fraction

EX) $3^{-2}=\frac{1}{3^2}$
$=\frac{1}{9}$

EX) $\frac{4}{2^{-1}}=\frac{4\bullet 2^1}{1}$
$=8$

EX) $\frac{2^{-3}}{3^{-1}}=$

$\frac{3^1}{2^3}=\frac{3}{8}$

EX) $\frac{2\bullet 4^{-1}}{3^2}=$

$\frac{2}{4^1\bullet 3^2}=\frac{2}{4\bullet 9}=\frac{2}{36}=\frac{1}{18}$

Estimation

EX) Evaluate the following expression using estimation

$$3.002\bullet 19.99+145.\overline{4}$$

$$\approx 3\bullet 20+145$$
$$\approx 60+145$$
$$\approx 205$$

Polynomials

Adding or Subtracting: Combine ***LIKE TERMS*** *(same variable with same exponent)*

Like Terms: $2x^3,\ 3x^3, -4x^3$ **NOT** Like Terms: $3x^3,\ 3y^3,\ 3x,\ 3y$

EX) Add $2x^3+3x^3-4x^3$

= $(2+3-4)x^3$

= x^3

EX) Subtract $\left(2xy+x^2-y^2\right) - \left(3y^2+x^2-9xy\right)$

$=2xy+x^2-y^2 - 3y^2-x^2+9xy$

$= 2xy+9xy+x^2-x^2-y^2 - 3y^2$

$=11xy-4y^2$

Multiplying: Multiply **all** terms from one expression with all terms from the other.

Remember x^3 means $x \bullet x \bullet x$
x^4 means $x \bullet x \bullet x \bullet x$
$So...x^3 \bullet x^4 = x^7$

EX) Multiply $\left(x-4\right)\left(2x+3\right)$ = $2x^2+3x-8x-12$ = $2x^2-5x-12$

EX) If A= $2x^3-3x+1$ & B= x^3-3

Find A • B and simplify by adding/subtracting like terms

$=\left(2x^3-3x+1\right)\left(x^3-3\right)$

$=2x^3 \bullet x^3+2x^3 \bullet \left(-3\right)+\left(-3x\right) \bullet x^3+\left(-3x\right) \bullet \left(-3\right)+1 \bullet x^3+1 \bullet \left(-3\right)$

$=2x^6-6x^3-3x^4+9x+x^3-3$

$=2x^6-3x^4-5x^3+9x-3$

Factoring

Factoring: Determining what polynomials were multiplied together

If $(x-4)(x+9)=x^2+9x-4x-36$ **then …….**

If asked to factor $x^2+5x-36$ **the answer is** $(x-4)(x+9)$

If $2x(x^3-4x) = 2x^4-8x^2$ **then…… if asked to factor** $2x^4-8x^2$ **the answer is** $2x(x^3-4x)$

EX) Factor $x^2 - 10x + 24$

$(x \pm number)(x \pm number)$ because x times x is x^2 when expressions are "foiled out"

$(x-6)(x-4)$ because "-6" and "-4" equal 24 when MULTIPLIED **AND** equal -10 when ADDED!

EX) Factor each expression

A) $x^2 - 9$ (no middle(linear) term so two numbers that add up to "zero")

$(x+3)(x-3)$

B) $x^2 - 5x - 24$

$(x+3)(x-8)$

C) $4x^3y - 2xy^2$ **(can't be a reverse of foiling so think a "reverse" of distributing a number called "common factoring")**

$2xy(2x^2 - y)$

D) $2x^2 + 4x - 30$ (Now use both methods by common factoring then foil factoring)

$2(x^2 + 2x - 15)$

$2(x+5)(x-3)$

Dividing Polynomials (simplifying rational expressions)

Since $\frac{3+5}{5}=\frac{8}{5}$ we **CANNOT** reduce(cancel) the two 5's when values are added or subtracted! Since $\frac{3\bullet 5}{5}=\frac{15}{5}=3$ notice you **CAN** reduce(cancel) the two 5's when values are multiplied!

So…. **FACTOR** (which creates multiplication) to simplify rational expressions!

EX) Simplify each expression

$$\frac{4x-2xy}{xy}$$

$$=\frac{2x(2-y)}{xy}$$

$$=\frac{2x(2-y)}{xy}$$

$$=\frac{2(2-y)}{y} \text{ or } \frac{4-2y}{y}$$

$$\frac{x^2+x-42}{(x+7)}$$

$$\frac{(x+7)(x-6)}{(x+7)}$$

$$\frac{\cancel{(x+7)}(x-6)}{\cancel{(x+7)}}$$

$$(x-6)$$

$$\frac{4x^2-25}{(2x+5)}$$

$$\frac{(2x+5)(2x-5)}{(2x+5)}$$

$$\frac{\cancel{(2x+5)}(2x-5)}{\cancel{(2x+5)}}$$

$$(2x-5)$$

Linear Equations

Solve linear equations by using inverse operations. Remember we ADD/SUBTRACT to both sides first, then MULT/DIV next to solve for a variable.

EX) Solve each equation for ***x***.

$2x-6=20$	$\frac{x}{4}+9=1$	$-2x+3y=40$
$2x=26$	$\frac{x}{4}=-8$	$=-2x=40-3y$
$x=13$	$4\bullet\frac{x}{4}=-8\bullet4$	$=x=\frac{40-3y}{-2}$ or $x=-20+\frac{3}{2}y$
	$x=-32$	

"PER" is a commonly used word in application problems and signifies an expression containing *multiplication*.

EX) Joe has $1000 in his bank account and withdraws $40 **per** week. Write a linear expression for this situation and use it to determine how many weeks it will take to have a balance of $80. Let "B" = Balance and "W" = weeks.

$40 **per** week = 40**W**

"withdraws" means SUBTRACTION

So… B = 1000 - 40w Now answer the question!

$$80=1000-40w$$
$$-920=-40w$$
$$\frac{-920}{-40}=w$$
$$23=w$$

So in **23** weeks the balance will be $80

When two variables are used, it is not uncommon to give the answer as an ordered pair.

If y = 2x + 3. then (1, 5) **is a solution**. Meaning, when x = 1 then y = 5.

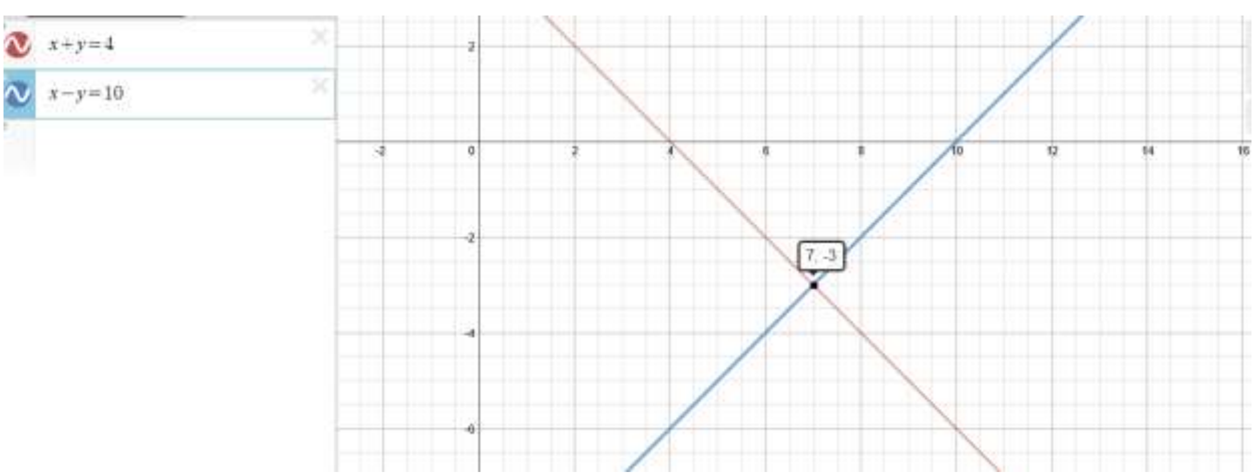
EX) If $A = \pi r^2$ is the area of a circle given its radius, what does the ordered pair (5, 25π) represent?

Since 5 is the first coordinate value it represents the independent variable (variable on the right of the equal sign) and 25π is the second coordinate value it represents the dependent variable (variable on the left of the equal sign) the following is true: **WHEN THE RADIUS IS 5 THE AREA IS 25π**

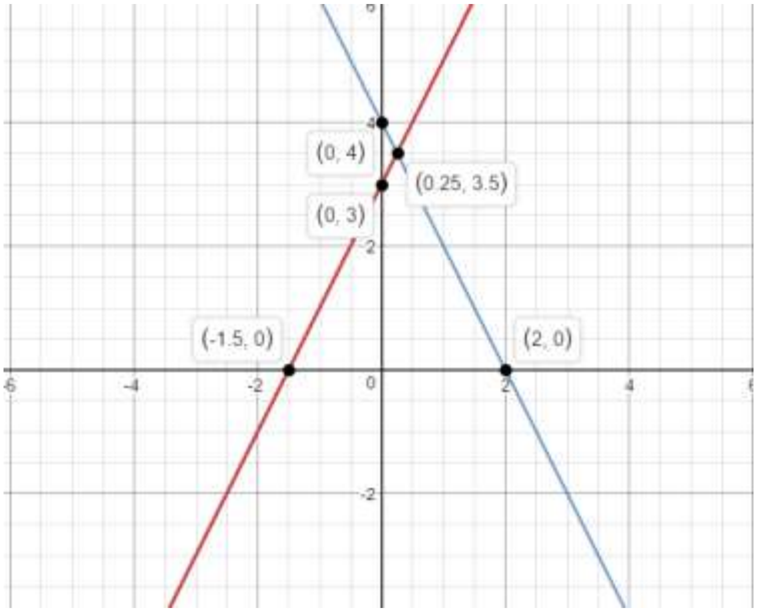
Systems of Linear Equations

System of Equations: two or more equations in the same variables

If $\begin{matrix} x+y=4 \\ x-y=10 \end{matrix}$ is a system of equations, then (7, -3) is its **solution** because it makes **BOTH** equations TRUE. This can be represented by the intersection of the two equations when graphed together.



EX) Use the graph to determine the solution to the system $\begin{matrix} y = 2x + 3 \\ 2x + y = 4 \end{matrix}$



(.25, 3.5)

Systems can be solved algebraically by **elimination** (**adding the two equations together**) or **substitution** (**substituting the contents of one equation in for that variable in the other equation**).

ELIMINATION

EX) Solve the system

$$\begin{matrix} 3x - 4y = 26 \\ x + 2y = 2 \end{matrix}$$

Since adding DOES NOT make x OR y CANCEL OUT you must multiply both sides of an equation so **IT DOES!**

$$\begin{matrix} 3x - 4y = 26 \\ 2 \bullet (x + 2y) = (2) \bullet 2 \end{matrix} \quad = \quad \begin{matrix} 3x - 4y = 26 \\ + \; 2x + 4y = 4 \\ \hline \end{matrix}$$

$$= \quad 5x + 0y = 30$$

$$= \quad 5x = 30$$

so $x=6$ (but this is only the x-value that makes both true). Now find the y-value.

If one of the equations is $x+2y=2$ and $x=6$ then $6+2y=2$. Solving for y gives you $2y=-4$ which means $y=-2$.

So, the solution to the system $\begin{array}{l} 3x-4y=26 \\ x+2y=2 \end{array}$ is **(6, -2)**.

EX) Solve $\begin{array}{l} x+y=4 \\ 2x+y=14 \end{array}$ by elimination.

$$\begin{array}{l} x+y=4 \\ -1\bullet(2x+y)=(14)\bullet(-1) \end{array}$$

$$\begin{array}{rl} & x+y=4 \\ + & -2x-y=-14 \\ \hline & -x+0=-10 \\ & x=10 \end{array}$$

If $x+y=4$ and $x=10$

Then $10+y=4$

$y=-6$

Solution: (10, -6)

SUBSTITUTION

EX) Solve the system

$$\begin{array}{l} 3x-4y=26 \\ x=2-2y \end{array}$$

$$\begin{array}{l} 3x-4y=26 \\ x=2-2y \end{array}$$

Since one of the equations is **SOLVED for a variable** simply substitute its contents into the other equation.

$3(2-2y)-4y=26$ and solve for the variable remaining.

$6-6y-4y=26$

$6-10y=26$ = $-10y=20$

so $y=-2$ (but this is only the y-value that makes both true). Now find the x-value.

If one of the equations is $x=2-2y$ and $y=-2$ then $\frac{675}{3}=225$.

Solving for x gives you $x=2-(-4)$ which means $x=6$.

So, the solution to the system $\begin{matrix} 3x-4y=26 \\ x=2-2y \end{matrix}$ is **(6, -2)**.

EX) Solve $\begin{matrix} y=4-x \\ x-y=10 \end{matrix}$ by substitution.

If $x-y=10$

Then $x-(4-x)=10$

$x-4+x=10$

$2x-4=10$

$2x=14$

$x=7$

If $y=4-x$ and $x=7$

Then $y=4-7$

$y=-3$

Solution: (7, -3)

Sometimes word problems turn into systems of equations

EX) Ryder truck rental costs a flat fee of $40 plus 20 cents **per** mile. U-Haul truck rental costs a flat fee of $30 plus 45 cents per mile. **Write a system of equations** where “C” represents cost and “m” represents miles. If the ordered pair (40, 48) is a solution to this system explain what this means.

Ryder: C = 40 + .20m

U-Haul: C = 30 + .45m

So, explain what “real world thing” is 40 when the other “real world thing” is 48!

Since **40** is the first coordinate value it represents the independent variable (variable on the right of the equal sign) and **48** is the second coordinate value it represents the dependent variable (variable on the left of the equal sign) the following is true:

Which rental company you choose depends on how many miles you need to drive it as the cost for each changes.

But, BOTH trucks when driven 40 miles cost 48 dollars because it is the solution to the system and makes both equations true!

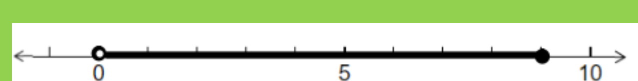
Linear Inequalities

Solutions to linear inequalities (in one variable) are **INFINITE (so ALL answers cannot be listed)** and are commonly graphed on a number line. A solid dot includes the border value and an open dot does not.

EX) Graph each inequality below on a number line.

$x \le 4$ or $x > 7$

$0 < x \le 9$ (think between)



EX) Solve each inequality and graph the solution set. **Remember to change the order (inequality sign) when multiplying or dividing by a negative number** otherwise, the answer will no longer be true.

$5 \le 3x - 4 < 17$

$9 \le 3x < 21$

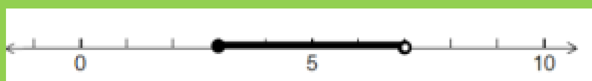
$3 \le x < 7$

$-2x \le 4$ or $-4x - 10 > 6$

$x \ge -2$ or $-4x > 16$

$x \ge -2$ or $x < -4$

Now graph solution set!



Absolute Values and Inequalities

If $|x| = 10$ then x = 10 or -10

So…If $|2x+4| = 10$ then **2*x* +4 =10 or 2*x* +4 = -10** meaning x = 3 or -7

EX) So… now solve $|2x+4| < 10$

Solve $2x + 4 < 10$ along with $2x + 4 > -10$

$2x < 6$ AND $2x > -14$

$x < 3$ $x > -7$

Graph these two answers:

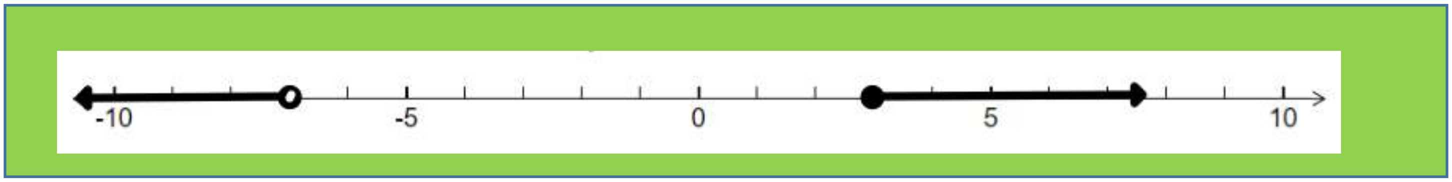
Since these graphs OVERLAP, the answer is the **COMMON** graphed region making the answer $-7 < x < 3$

EX) Now solve and graph $|2x+4| \geq 10$ and notice the difference in the solution when changing the order(inequality symbol).

Solve $2x+4 \geq 10$ along with $2x+4 < -10$

$2x \geq 6$ $\quad$ $2x < -14$

$x \geq 3$ $\quad$ $x < -7$



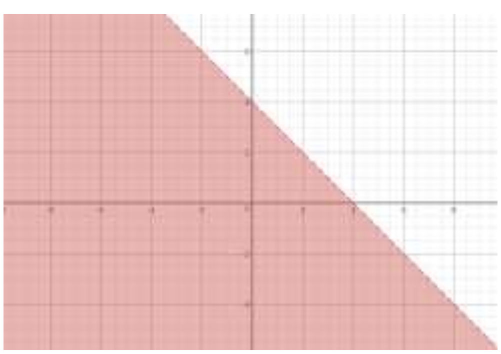
So the final answer is $x < -7 \text{ or } x \geq 3$

System Linear Inequalities

Since a system of linear inequalities are graphed on the coordinate plane (because the linear inequalities contain TWO variables) the solution represents the INFINITELY **COMMON** SHADED region of ordered pairs.

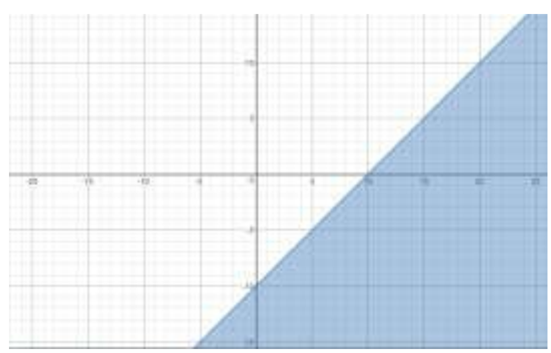
EX) Solve $\begin{array}{l} x+y<4 \\ x-y \geq 10 \end{array}$ by graphing!

$x+y<4$



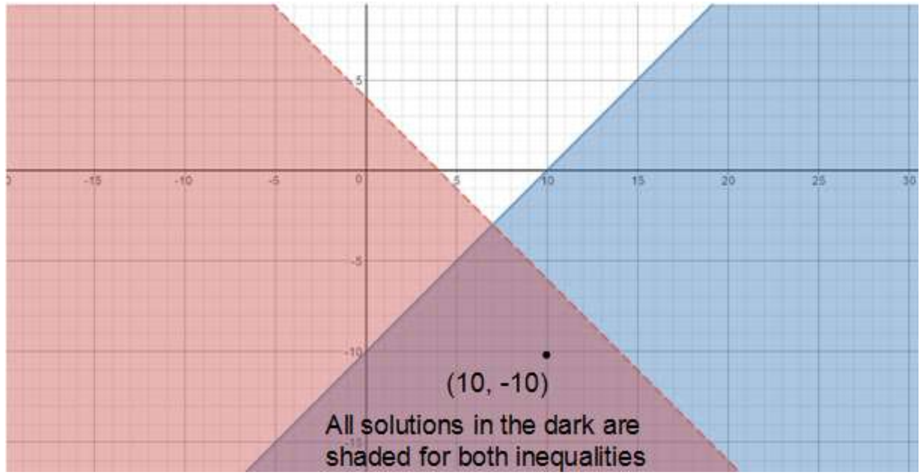
Notice **dotted** border and all ordered pairs in the shaded region such as (0, 0) make the inequality true!

$x-y \geq 10$



Notice **solid** border and all ordered pairs in the shaded region such as (15, 0) make the inequality true!

$x + y < 4$
$x - y \geq 10$



Ordered pairs in the **common shaded region** (such as (10, -10)) represents the **SOLUTION** since it makes **BOTH** inequalities of the system TRUE! Now find three more solutions to this system of inequalities.

Infinite answers such as: (5, -10), (0, -15), (15, -20)

Keystone Algebra I Review

(Module 2 Answers)

Patterns of Numbers

Continue the pattern for the next four values:

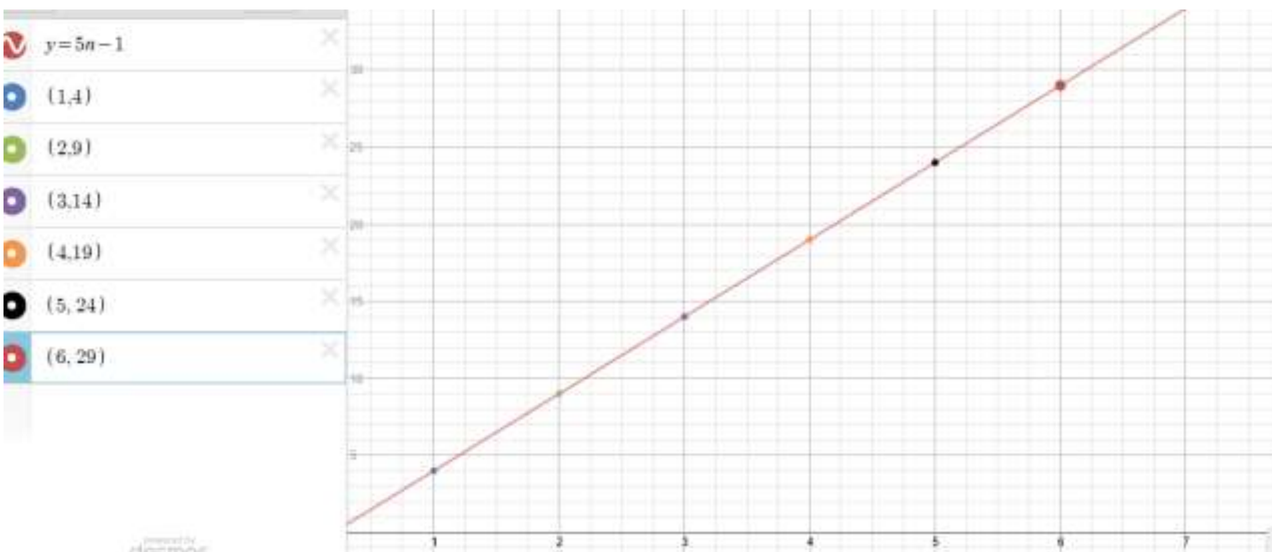
4, 9, 14, 19, ….

The variable "n" commonly represents what term you are on (ex: when n = 3, we are referring to the value of the third term which is 14).

So… the sequence 5n-1 could be used to describe the sequence of numbers. Find terms five through eight in the sequence above.

24, 29, 34, 39

The sequence from above 4, 9, 14, 19, …. can be represented **graphically**:



or by table:

x	y
3	14
4	19
5	24
6	29

$y = 5x - 1$ still fits the formula to match the table of values

EX) Write the first four terms for the sequence of numbers represented by $n^2 - 3$ and make a table to represent this sequence. -2, 1, 6, 13

Relations and Functions

Function: a relation between x and y such that **EACH x-coordinate**(Domain) is paired with **ONLY ONE y-coordinate**(Range). So…. If a set of ordered pairs has duplicate x-values, it cannot be a function. Its graph, will then **fail** the Vertical Line Test(VLT) as a vertical line intersects more than once indicated the relation is NOT a function.

EX) Determine the Domain, Range of each relation and determine if it is a function.

A) (-2, 5), (3, -1), (0,5), (4, -1)

Domain: {-2, 3, 0, 4}

Range: {-1, 5}

NO DUPLICATE x-values

So this IS a function!

B)

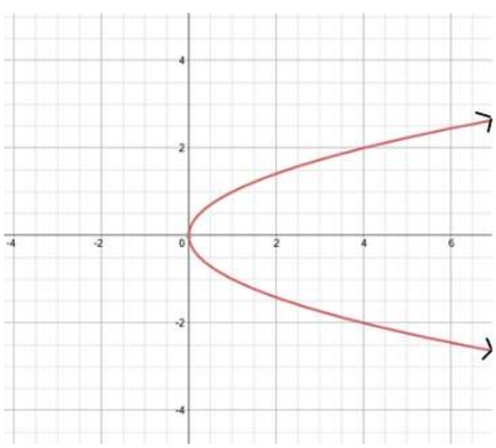
x	y
1	1
3	2
5	3
3	4

Domain: {1, 3, 5}

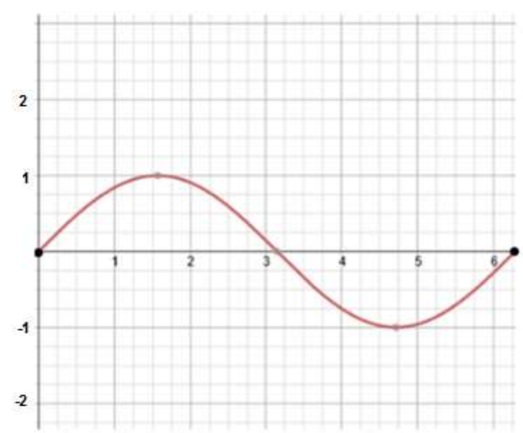
Range: {1, 2, 3, 4}

"3" is paired with both "2" and "4" so this **IS NOT** a function

C)



D)



Domain: $x \geq 0$

Range: All real numbers ($-\infty < y < \infty$)

NOT A FUNCTION (fails VLT)

Domain: $0 \leq x \leq 6.1$

Range: $-1 \leq x \leq 1$

Linear Functions

Linear Function: A function of the form **y = mx + b** where **"m"** represents the ***rate of change***(slope) and **"b"** represents the initial value for y (**y-intercept**) when x is zero. Note: the slope is sometimes indicated using the word "**per**"!

EX) What is the slope and y-intercept for each linear function

y= -3x +5	2x = 8 - y(**solve for y first**)	3x-4y=24 (**solve for y first**)
Slope: -3	y = -2x +8	-4y= -3x+24
y-int: 5	Slope: -2 y-int: 8	y = 3/4x - 6
		Slope: 3/4 y-int: -6

EX) A person drives 65 miles **per** hour. Write a linear function for the distance "D" someone drives after "h" hours.

D= 65h (notice "per hour" means "times h" in our expression)

Use this to determine how far a person drives after 5 hours.

D= 65 •h **D= 65 •5** **D= 325 miles**

EX) During the first week of December a machine for a cell phone manufacturer produced 3400 cell phones. The company decides to speed up the machine to help demand for the holiday season so that it could produce 3 cell phones per minute. Write a linear function for this situation where "**C**" represents the number of cell phones total for the month of December made after "**m**" minutes since it was speeded up. Use this to determine how many minutes are needed at this new production speed to produce a total of 25,000 cell phones for the month of December.

Hint: 3 per minute = **3m** and remember there were 3400 cell phone already produced prior to this new speed.

C = 3m + 3400

25000 = 3m + 3400 21600 = 3m so **7200 minutes**

Graphing & Finding Equations for Linear Functions

$$y = mx + b$$

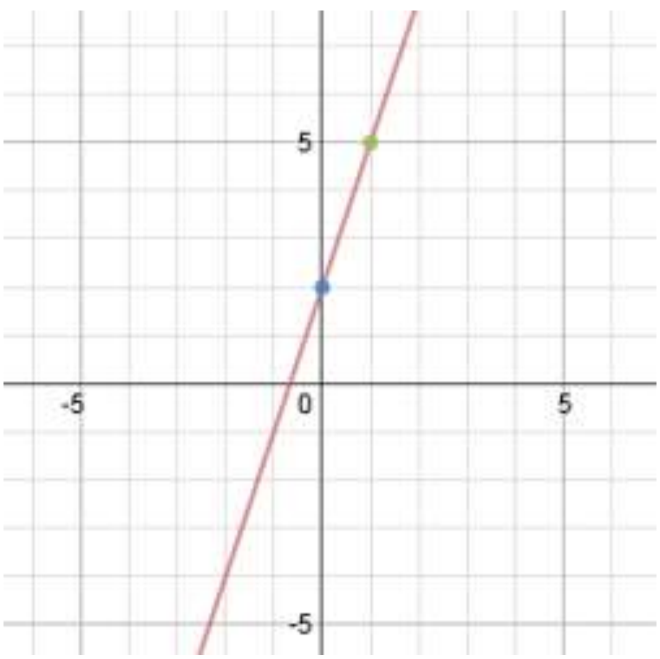
Slope(**m**) is determined by finding $\frac{\text{Change in } y \text{ value}}{\text{Change in } x \text{ value}} = \frac{\Delta y}{\Delta x}$ (*also called rate of change*)

up means **positive *y* change** and **down** means **negative *y* change**.
right means **positive *x* change** and **left** means **negative *x* change**.

y-intercept(**b**) is the **y-value** where the line **crosses the y-axis**.

EX) Determine the equation for each linear function graphed below.

A)

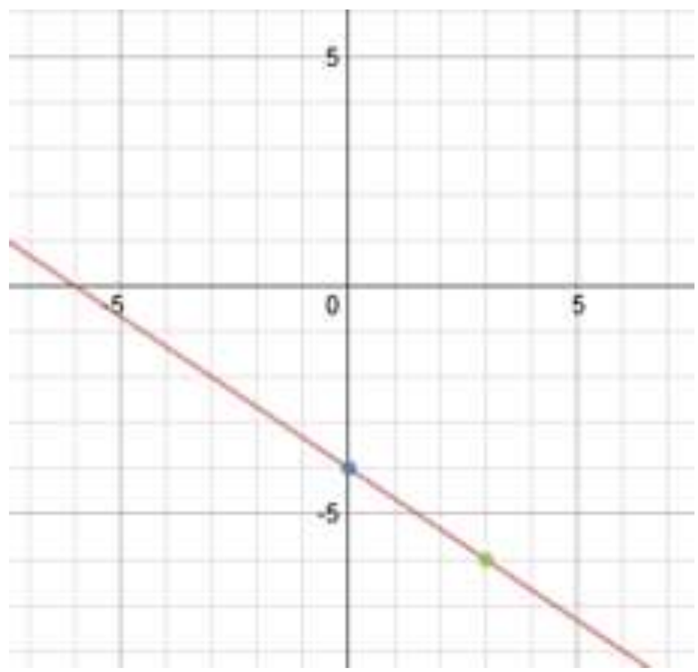


y-intercept is **2**

Slope is $\frac{up\ 3}{right\ 1} = \frac{+3}{+1} = \frac{3}{1}$ **= 3**

So… $y = 3x + 2$

B)



y-intercept is **-4**

Slope is $\frac{down\ 4}{right\ 6} = \frac{-4}{+6} = \frac{-2}{3}$ **=** $-\frac{2}{3}$

So… $y = -\frac{2}{3}x - 4$

EX) Find a linear equation for a function that passes through (-6, 5) and (6, 1)

Slope = $\frac{\textit{change} \text{ from 5 to 1}}{\textit{change} \text{ from -6 to 6}} = \frac{-4}{12} = \frac{-1}{3}$ which means $-\frac{1}{3}$

We do not have the **y-intercept** so we need to find it.

$y = -\frac{1}{3}x + b$

$1 = -\frac{1}{3} \bullet 6 + b$ (use either of the two points and plug in for x and y)

$1 = -2 + b$

$3 = b$

So… $y = -\frac{1}{3}x + 3$

EX) Find a linear function which models the table below.

x	y
-15	6
-6	0
3	-6
12	-12

Using (-15, 6) and (-6, 0)

Slope is $\frac{\textit{down } 6}{\textit{right } 9} = \frac{-6}{+9} = \frac{-2}{3} = -\frac{2}{3}$

We don't know the y-int. so must solve for it!

$y = -\frac{2}{3}x + b$

$0 = -\frac{2}{3} \bullet (-6) + b$ (using (-6, 0))

$0 = 4 + b$

$-4 = b$

So… $y = -\frac{2}{3}x - 4$

EX) The table below represents the square yards of grass(g) a person could mow after a certain amount of hours(t). Find the **Unit Rate** (square yards mowed **per** hour) by finding the slope. Use this to write an equation and determine the time needed to mow 12,000 square yards. Remember, although not listed, (0,0) must be an ordered pair since ZERO sq. yards are mowed after ZERO hours.

t (hours)	$g\ (yd^2)$
2	450
5	1125
8	1800

Slope: $\frac{675}{3} = 225$ **(this is the unit rate per hour)**

y-int: 0

Equation: g = 225h

So...12000=225h

H = 53.3 hours

Now answer the question.

Point-Slope Form for a Linear Equation: a linear function containing the slope "**m**" and ANY point (x_1, y_1) (not necessarily the y-intercept).

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) represents any coordinate on the line

EX) Find a linear equation for a function that passes through (-6, 5) and (6, 1).

Slope = $-\frac{1}{3}$

$y - 1 = -\frac{1}{3}(x - 6)$ **OR** $y - 5 = -\frac{1}{3}(x - (-6))$ = $y - 5 = -\frac{1}{3}(x + 6)$

Note: This is equivalent to the equation we have above in y = mx +b form.

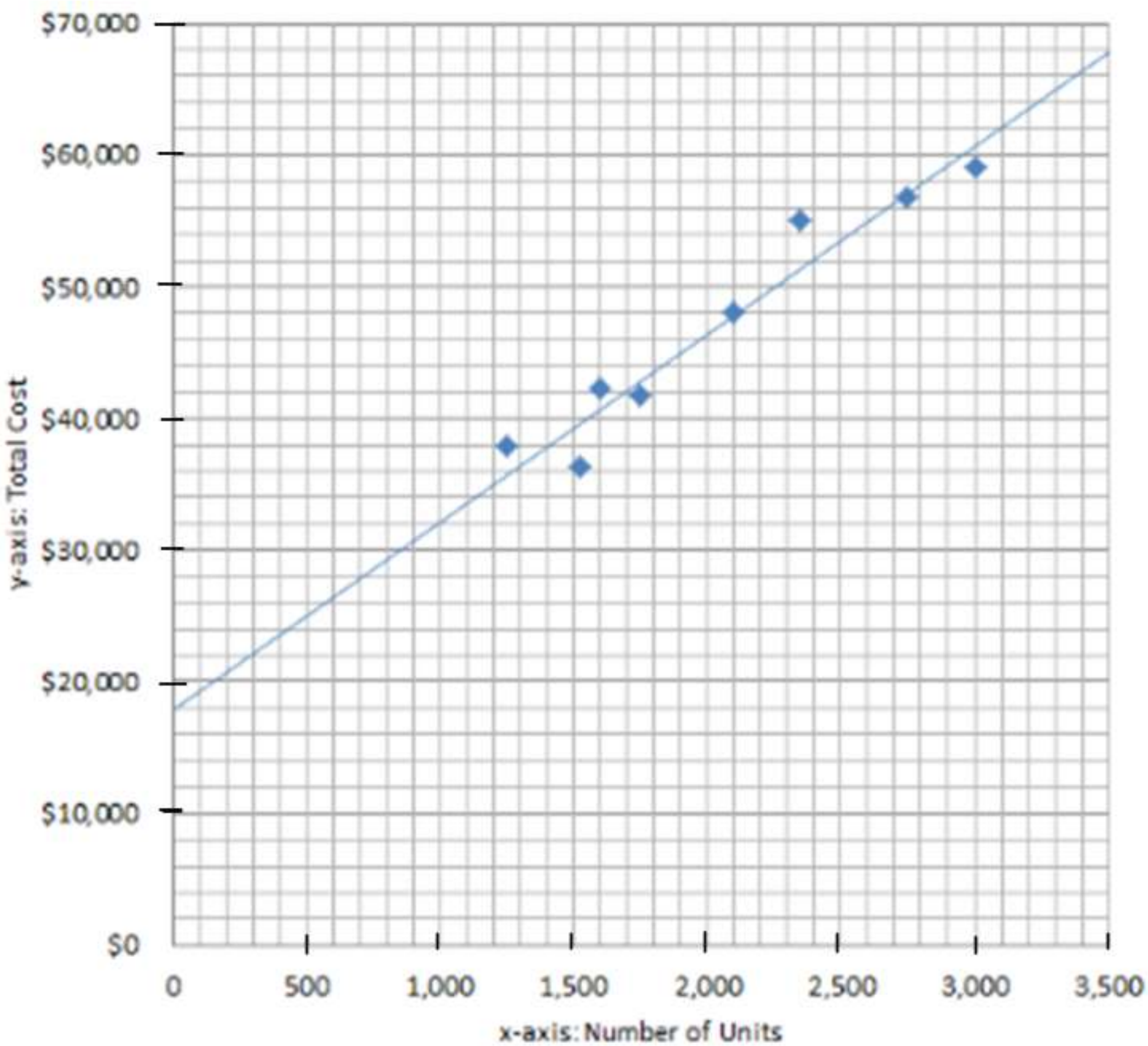
SUMMARY FOR WRITING EQUATIONS: it is easiest to use y = mx+b form when we KNOW the y-intercept (and don't need to solve for it). Otherwise, point slope form may be used since ANY POINT can be used for (x_1, y_1).

Lines of Best Fit

Real world data often **does not** graph a perfect line but **DOES** form a **LINEAR TREND**. So, we can draw a line of best fit and determine its linear function to use to make predictions!

Note: When trend forms a line with positive slopes it is known as a ***positive correlation***. When trend forms a line with negative slopes, it is known as a ***negative correlation***. If the data is too scattered to tell, it has NO CORRELATION thus no line of best fit can be made.

The line of best fit drawn below is used to determine the total cost(y) of making (x) units of goods for a certain company.



Note: every vertical line = 100 and every horizontal line = 2000

Ex) Use the line of best fit to estimate the cost of making 400 units.

$24,000 is the approximate y-value on the graph when x = 400

EX) Create a linear equation for this line of best fit and use it determine the cost of making 5000 units.

Line of best fit goes through (0, 18000) and (1400, 38000) and also note (0, 18000) is the y-intercept.

Write your equation in y = mx +b form and then use it to find y when x =5000.

y-intercept is **18000**

Slope is $\frac{up\ 20000}{right\ 1400} = \approx 14.3$

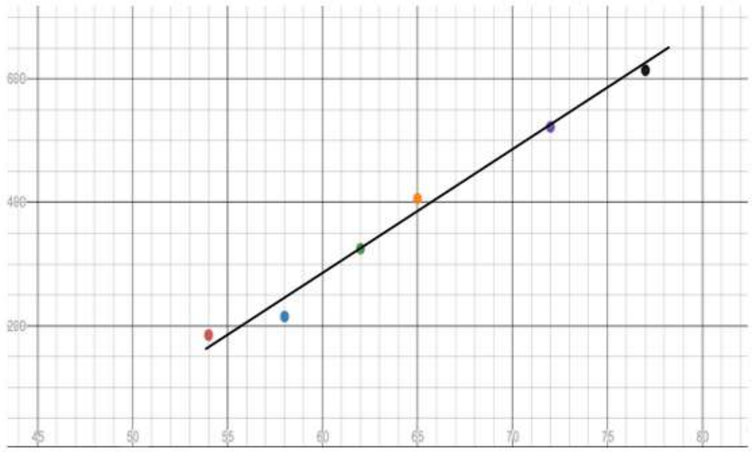
Equation: $y = 14.3x + 18000$

$y = 14.3 \bullet 5000 + 18000$

$y = \$89,500$

EX) Use the line of best fit to create a linear function for the data and use it to predict the amount of ice cream sales when the temperature is 90°F.

Ice Cream Sales vs Temperature	
Temperature °F	**Ice Cream Sales**
54°	$185
58°	$215
62°	$325
65°	$406
72°	$522
77°	$614



Using the points (62, 325) and (72, 522) create the equation of this line.

Slope is $\frac{up\ 197}{right\ 10} = \mathbf{19.7}$

No y-int. so using point-slope form:

$y-325=19.7(x-62)$ **OR** $y=19.7(x-62)+325$

Now solve for y when x = 90

$y=19.7(90-62)+325$

$y=19.7\bullet(28)+325$

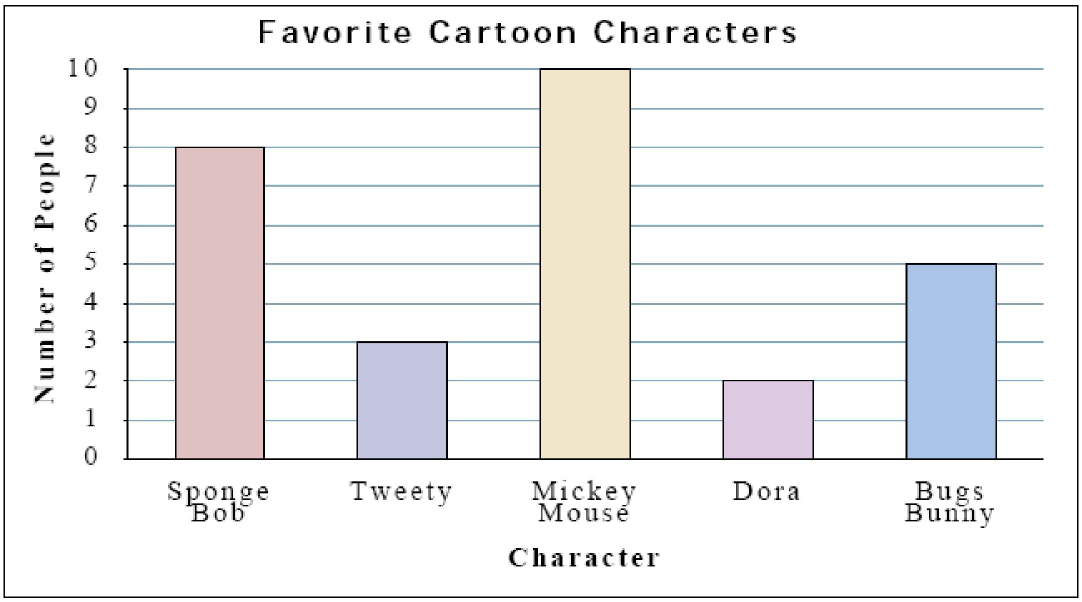
$y=876.6$

So, when the temperature is 90 °F ice-cream sales are approximately **$876.60**

Reading Bar Graphs and Circle Graphs

Remember, $\textbf{\textit{Percent}} = \frac{part}{whole} \bullet 100$

EX) Below is the bar graph of the poll results of people's the favorite cartoon character



A) Determine the percentage of people who prefer Sponge Bob as their favorite cartoon character.

Amount that prefer Sponge Bob = 8

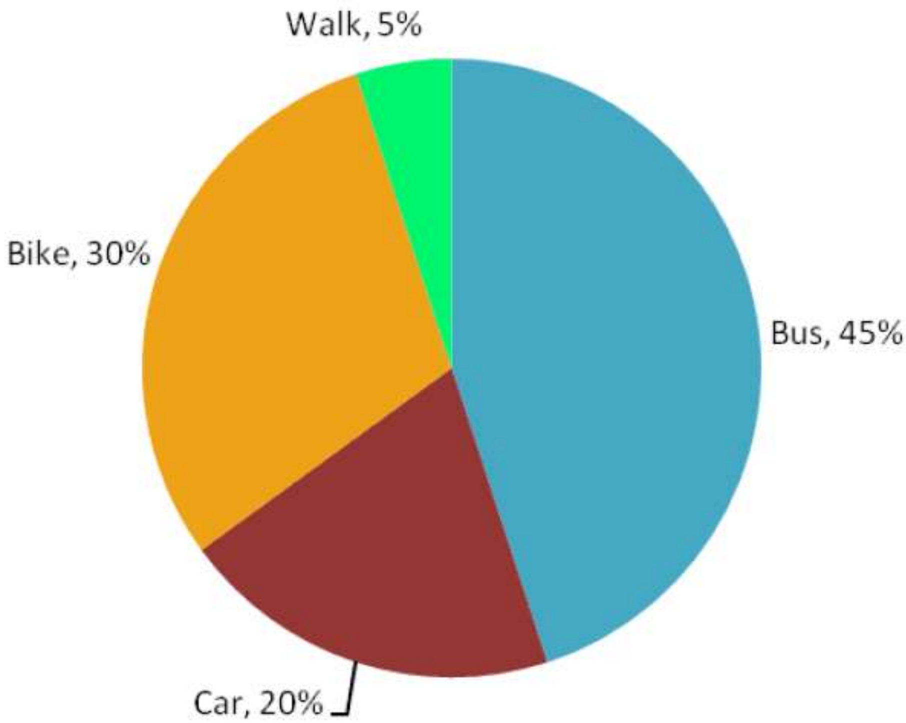
Total number of people in the poll = 28

Percentage of people who prefer Sponge Bob = $\frac{8}{28} \bullet 100 = 28.6\%$

B) Determine which cartoon character received the least amount of votes and calculate its percentage.

Dora with only 2 votes. So, $\frac{2}{28} \bullet 100 = 7.1\%$

EX) If 1530 students commute to Nazareth High School each day by various means (with percentages given below), determine the amount of students that walk, the amount of students that bike, the amount of students that take the bus and the amount of students that take the car.



Students that Walk = 5% so…. 5% of 1530 students walk

5% means 5/100 and "**of**" means **times**

$\frac{5}{100} \bullet 1530 = 76.5$ so about **76 students walk to school**!

Now find the amount that ride a bike, take the car, and take the bus to school.

30% ride their bike to school, 20% take a car and 45% take the bus

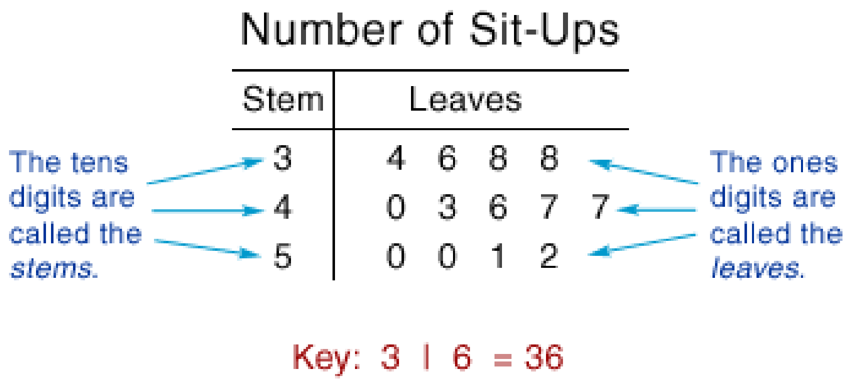
$\frac{30}{100} \bullet 1530 = 459$ so about **459 students ride their bike to school**!

$\frac{20}{100} \bullet 1530 = 306$ so about **306 students take a car to school**!

$\frac{45}{100} \bullet 1530 = 688$ so about **688 students take the bus to school**!

Range: High-Low **Mean:** average **Median:** middle value **Mode:** most frequent

EX) Use the stem and leaf plot for the number of sit ups completed in gym class to find the range, mean, median and mode of the data.



The above stem and leaf diagram represents the following list of numbers:

34 36 38 38 40 43 46 47 47 50 50 51 52

Range: 18 Mean: 44 Median: 46 Mode: 38, 47, 50

All appear three times!

Box & Whisker Plots

To create a box and whisker plot we need:

Minimum: Lowest Value

Maximum: Highest Value

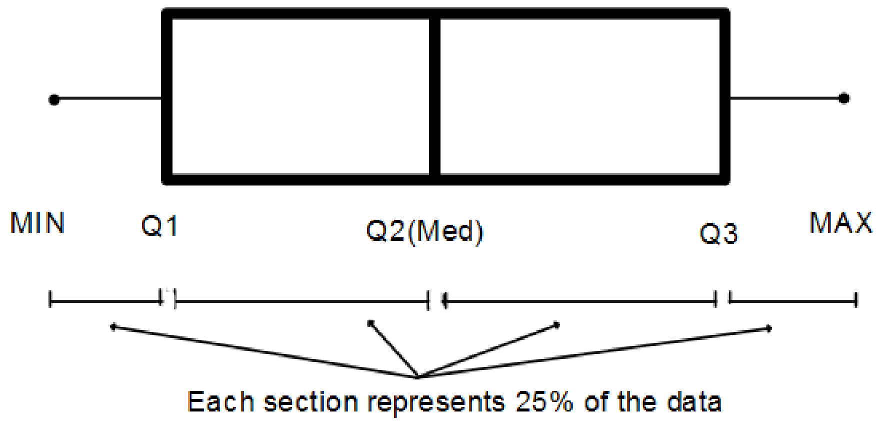
Median (also called Quartile 2(Q2): Middle Value (or average of the two middle values if there isn't one)

Quartile 1(Q1): Median (or average of the two middle values if there isn't one) of the Lower half

Quartile 3(Q3): Median (or average of the two middle values if there isn't one) of the Upper half

Interquartile Range: Range of the values from Q1 to Q3 so (**Q3-Q1**)

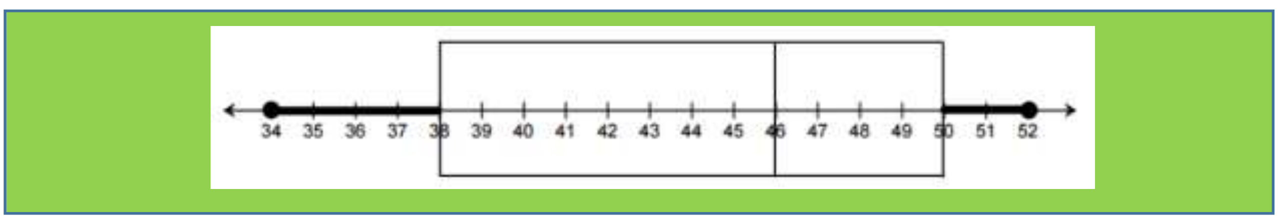
Plot the Min, Q1, Q2(median), Q3 and the Max to create your box & whiskers plot creating 4 distinct regions all of which represent 25% of the data!



EX) Create a box and whisker plot for the data set above containing the number of sit ups completed in gym class.

Minimum: **34** **Maximum:** **52** **Median:** **46** **Q1:** **38** **Q3:** **50**

Interquartile Range: 12 (so the box should be 12 units in length)



EX) Use your box and whisker plot to answer the following questions:

A) The least amount of sit-ups completed was 34

B) 75% of the students did at least 38 sit-ups

C) 25% of the students did at least 50 sit-ups

D) 50% of the students did less than 46 sit-ups

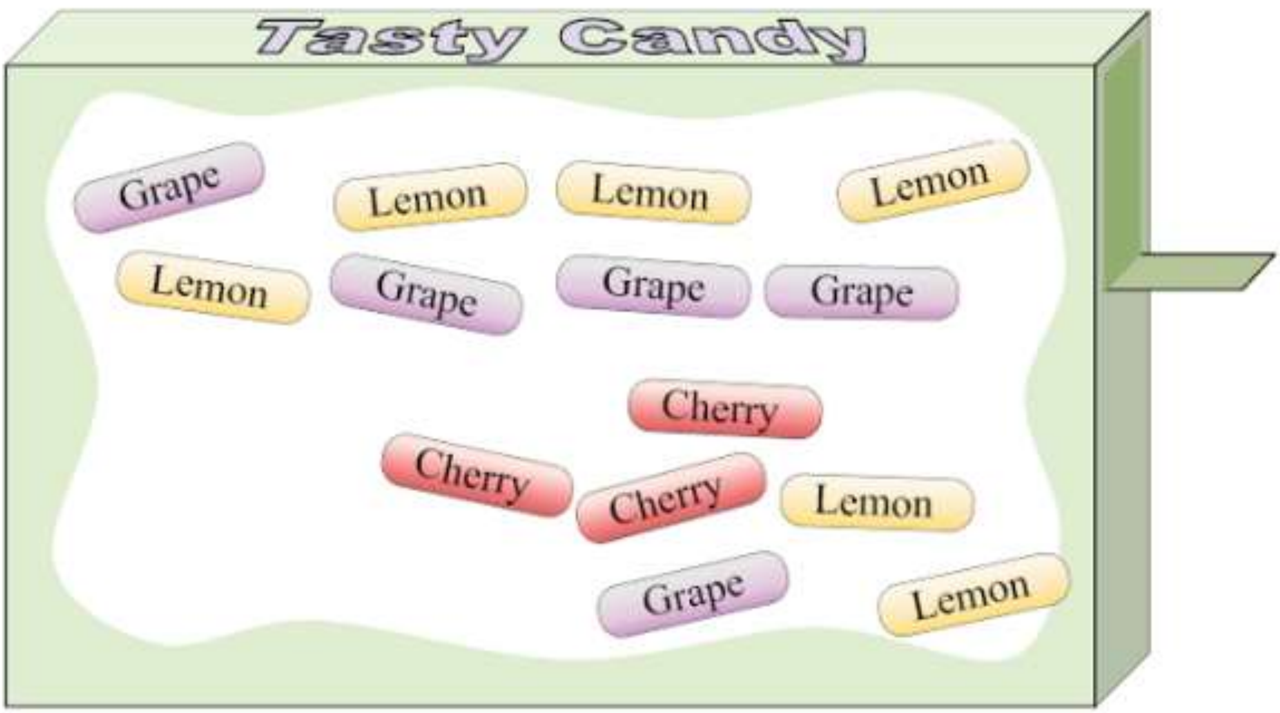
Probability

$$\text{Probability} = \frac{\text{Number of favorable outcomes}}{\text{Total possible outcomes}}$$

Add probabilities when the word **"or"** is between events.

Multiply probabilities when **events happen consecutively** and you are asked to determine probability of them **happening together**. Note: It is important to know if the first event effects the second in terms of total possible outcomes.

EX) Use the box of candy given below to answer the probability questions



A) Find the probability of picking a lemon candy.

$$\text{Probability} = \frac{\text{Number of Lemon Candies}}{\text{Total Number of Candies}} = \frac{6}{14} = \frac{3}{7}$$

B) Find the probability of **not** picking a grape candy.

$$\text{Probability} = \frac{\text{Non-Grape Candies}}{\text{Total Number of Candies}} = \frac{9}{14}$$

C) Find the probability of picking a grape **or** a cherry candy.

$$\text{Probability} = \frac{\text{Grape Candies} + \text{Cherry Candies}}{\text{Total Number of Candies}} = \frac{8}{14} = \frac{4}{7} \text{(when reduced)}$$

D) Find the probability of picking a cherry candy without replacement **followed by** a lemon candy.

$$\text{Probability} = \frac{\text{Number of Cherry Candies}}{\text{Total Number of Candies}} \bullet \frac{\text{Number of Lemon Candies Remaining After First is Picked}}{\text{Total Number of Candies Remaining After First is Picked}}$$

$$= \frac{3}{14} \bullet \frac{6\text{(since the first one picked was not lemon)}}{13\text{(since one was picked and not replaced)}}$$

$$= \frac{18}{182} = \frac{9}{91}$$

E) Find the probability of picking a lemon candy, replacing it, followed by a grape candy.

$$\text{Probability} = \frac{\text{Number of Lemon Candies}}{\text{Total Number of Candies}} \bullet \frac{\text{Number of Grape Candies}}{\text{Total Number of Candies(because first pick was replaced)}}$$

$$= \frac{6}{14} \bullet \frac{5}{14}$$

$$= \frac{30}{196}$$

$$= \frac{15}{98} \text{ (when reduced)}$$