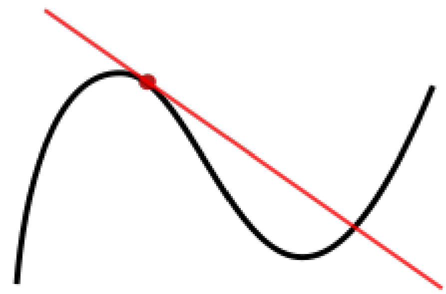
The Limit Definition of Derivative

w-up: Write an equation of a line traveling through (-3, 4) with slope of ½.

Tangent line: a straight line that "just touches" the curve at that point. (looks like a see-saw)

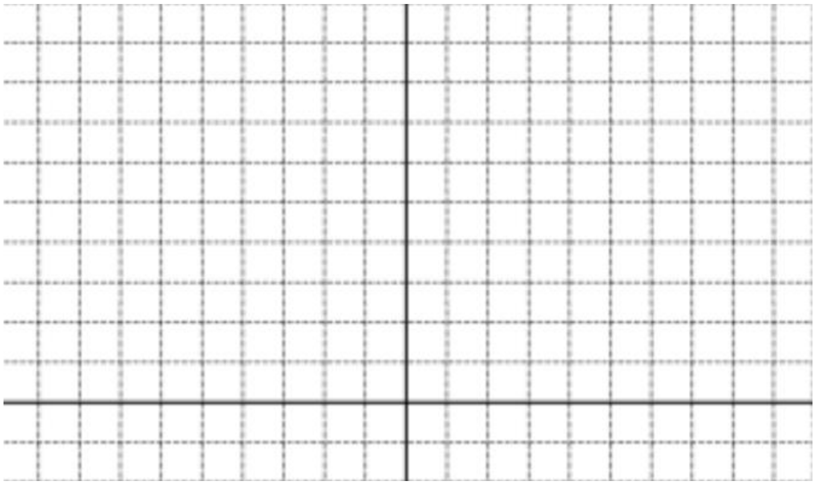


Draw five more tangent lines on the curve above.

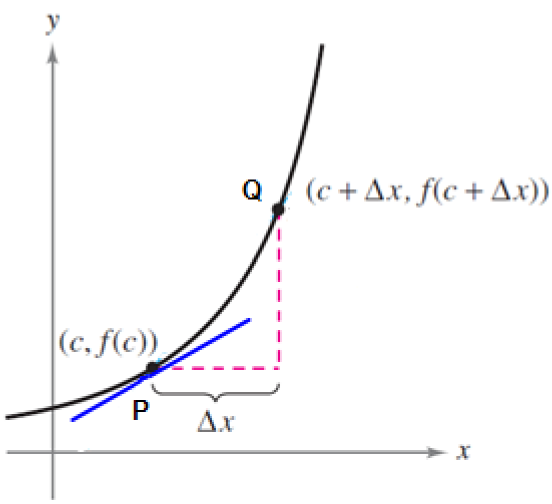
The slope of the tangent line represents the "steepness" of any curve at any point and we call this steepness the **Instantaneous Rate of Change** of a function at any point.

Question: What kind of functions is the instantaneous rate of change constant(always the same)?

EX) Sketch the graph of $y=\frac{1}{4}x^2$ on graph paper and estimate the slope of the tangent line at x = 2, x = 4 and x =-1



Finding the exact slope of a tangent line using limits



Let point P be any point where the Rate of Change (slope) is to be found.

Let point Q be any point on the function Δx away from the x-coordinate of point P.

Point P $\left(c, f(c)\right)$ **Point Q** $\left(c+\Delta x, f(c+\Delta x)\right)$

Write an expression using these coordinates to find the slope of $\overline{PQ}$.

Note: the closer point Q is to point P(so as Δx gets closer to zero) the closer the slope of the secant is to the actual slope of the tangent line at point P.

Slope of the Tangent line = $\lim_{\Delta x \to 0} \frac{f(c+\Delta x)-f(c)}{\Delta x}$

where c is the x-value constant we want to find slope at!

So, $\lim_{\Delta x \to 0} \frac{f(2+\Delta x)-f(2)}{\Delta x}$ using the function $f(x)=\frac{1}{4}x^2$ will find the slope of the tangent line to $f(x)$ at the point (2,1). Find the exact value of the slope at this point.

Evaluate $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ *without* substitution for the x-value. This will result in an algebraic expression instead of a value. This will serve as a "slope finding formula" for finding the slope of a tangent line at **any** x-value!

So... $\lim_{\Delta x \to 0} \frac{\frac{1}{4}(x+\Delta x)^2-\frac{1}{4}x^2}{\Delta x}$ finds the slope finding formula to use to find the slope at ANY x-value for the function $f(x)=\frac{1}{4}x^2$

Use this "slope finding formula" to find the slope of the tangent line to

$f(x)=\frac{1}{4}x^2$ at x = 4 and x = -1

This "slope finding formula" is known as the **DERIVATIVE**

Derivative: Functional expression which will find the Rate of Change(slope of the tangent line) for any function at any point

Note: The process of finding a derivative is called "differentiation"

Notation

$f'(x)$ (read f prime of x)

$\frac{dy}{dx}$ (read dy, dx)

y'

$\frac{d}{dx}f(x)$

EX) Find the equation of the tangent line to $f(x) = \frac{1}{4}x^2$ at x = -3

Alternate Forms for the Limit Definition of Derivative

Using "h" instead of Δx	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$	**finds derivative at any x-value**
Finds slope at a singular x-value (which is the x-value the limit is approaching)	$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$	**finds the slope of $f(x)$ at $x = a$**

AP EXAMPLES

Explain the meaning of each limit(you are not actually finding the value).

A) $\lim_{\Delta x \to 0} \frac{\cos(x+\Delta x)-\cos x}{\Delta x}$

B) $\lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4}+h\right)-\tan\frac{\pi}{4}}{h}$

C) $\lim_{x \to 3} \frac{2x^3-54}{x-3}$

D) $\lim_{x \to a} \frac{\ln(x)-f(a)}{x-a}$