Limits to Infinity

w-up: Give the equation of the horizontal asymptote (if any) for each function. If you do not remember how to find it, use the graphing calculator to help and/or verify your answer.

$A)\quad f(x)=\frac{3x+1}{x-1}$ $\qquad B)\quad f(x)=-\frac{2}{x^2+1}$ $\qquad C)\quad f(x)=\frac{2x^2-1}{x+4}$

Finding Horizontal Asymptotes (*for Rational Functions Containing Polynomials*)

Degree of Numerator = Degree of Denominator

$$y=\frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$$

Degree of Numerator < Degree of Denominator the H.A.

$$y=0$$

Degree of Numerator > Degree of Denominator the H.A.

NO Horizontal Asymptote(but will have a slant/other asymptote)

Reminder: Horizontal Asymptotes are NOT values which make a function undefined and CAN contain points from the function.

Limit to infinity: The *y*-value a graph approaches as the *x*-values get infinitely large ($+\infty$) or infinitely small ($-\infty$).

Find the $\lim_{x\to\infty} f(x)$ for the above functions(A-C).

Find the $\lim_{x\to-\infty} f(x)$ for the above functions(A-C).

A limit to infinity **is** the horizontal asymptote for Rational Functions!

ZERO is the limit to infinity of any function containing a fraction where the **denominator increases without bound** but the numerator DOES NOT. Note: also true if the **denominator increases without bound** QUICKER than the numerator increases without bound (happens when exponential functions are in the numerator and denominator yielding $\frac{\infty}{\infty}$ or $\frac{-\infty}{-\infty}$).

EX)

A) $\lim_{x\to\infty}\frac{2^x+5}{3^x+1}$

B) $\lim_{x\to-\infty}\frac{e^x+5}{2^x+1}$

C) $\lim_{x\to\infty}\frac{e^x+5}{2^x+1}$

Rational Functions Containing Exponential and Polynomial Functions

When a rational function contains an exponential (with b >1) **and** a polynomial function that both grow without bound, know that the **Exponential Function will eventually grow QUICKER than the Polynomial Function**.

A) $\lim_{x\to\infty}\frac{3x^2}{2^x}$

B) $\lim_{x\to\infty}\frac{2^x}{3x^2}$

C) $\lim_{x\to-\infty}\frac{2^x}{3x^2}$

EX) Use Properties of Limits for Composite Functions to Evaluate each Limit

A) $\lim_{x\to\infty} x^{1/x}$

B) $\lim_{x\to 3^-} 5^{2/(x-3)}$

Special Case(Oscillation)

Find $\lim_{x \to \infty} \sin x$

EX) $\lim_{x \to \infty} \frac{\sin x}{x}$

NOTE: although zero is the limit, it is NOT a horizontal asymptote (as $y = \sin x$ oscillates)! We call this non-asymptotic behavior.

Infinite Limits when *Limits are taken to Infinity*(so, the limit DNE but can be denoted with $\pm\infty$)

EX) $\lim_{x \to \infty} x^3 - 1{,}000{,}000{,}000{,}000x - 1{,}000{,}000{,}000{,}000$

HIGHEST POWER WINS!

EX) $\lim_{x \to -\infty} \frac{3x^2 - 2x + 1}{x + 4}$

Determine the sign of infinity by dividing signs of numerator and denominator!

Use equivalent forms of algebra to evaluate the following limits

EX) $\lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$

EX) $\lim_{x \to -\infty} \frac{3x - 2}{\sqrt{2x^2 + 1}}$