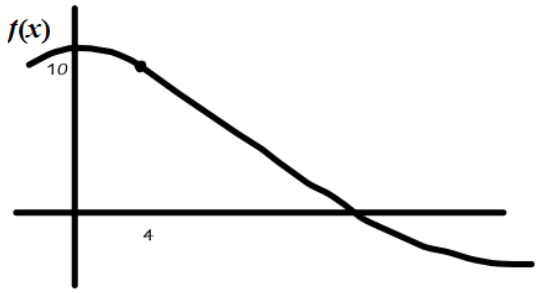
Linearization and Tangent Line Approximation

Tangent Line Approximation: Using the tangent line to approximate a value of a function when the equation of that function is not known(or could be found)



$f(4)=10$

$f'(4)=-\frac{1}{2}$

EX: Approximate $f(4.1)$ using the tangent line drawn to $f(x)$ at $x = 4$ and determine if this is an overapproximation or underapproximation.

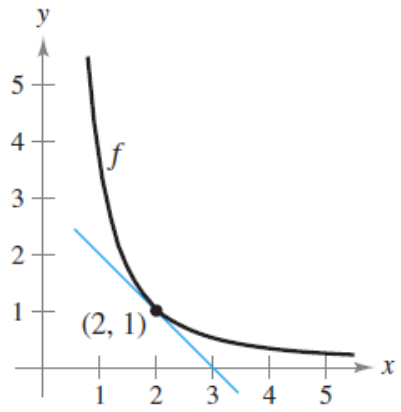
Since the y-value of the function at $x = 4.1$ is extremely close to the y-value of the tangent line at $x = 4$, evaluate the equation of the tangent line at $x = 4.1$.

NOTE: If graph is **concave down** at point of tangency, the estimation is an OVERAPPROXIMATION & If graph is **concave up** at point of tangency, the estimation is an UNDERAPPROXIMATION!

EX: Given $\frac{dy}{dx}=\frac{7}{\sqrt{x^2+7}}$ and $f(3)=\frac{3}{4}$, use linear approximation to estimate $f(2.9)$ and determine if this is an overapproximation or underapproximation.

Sometimes tangent lines are used to approximate a ZERO of the original function.

EX) Give an estimate for a zero of f using tangent line approximation.



AP Example

Graphing Calculator Allowed

Let f be the function given by $f(x) = x^2 - 2x + 3$. The tangent line to the graph of f at $x = 2$ is used to approximate values of $f(x)$. Which of the following is the greatest value of x for which the error resulting from this tangent line approximation is less than 0.5?

(A) 2.4 (B) 2.5 (C) 2.6 (D) 2.7 (E) 2.8