Integrate each function.

1a) $\int \frac{x}{x^2 - 3}\,dx$

$u = x^2 - 3,\ du = 2x\,dx$

$$\int \frac{x}{x^2 - 3}\,dx = \frac{1}{2}\int \frac{1}{x^2 - 3}(2x)\,dx$$
$$= \frac{1}{2}\ln|x^2 - 3| + C$$

1b) $\int \frac{1}{2x + 5}\,dx$

1c) $\int \frac{4x^3 + 3}{x^4 + 3x}\,dx$

1d) $\int \frac{\sec x \tan x}{\sec x - 1}\,dx$

1e) $\int_1^2 \frac{1 - \cos\theta}{\theta - \sin\theta}\,d\theta$

2a) $\int \frac{x^2 - 3x + 2}{x + 1}\,dx$

2b) $\int \frac{x^3 - 3x^2 + 5}{x - 3}\,dx$

2c) $\int \frac{x^4 + x - 4}{x^2 + 2}\,dx$

USE LONG DIVISION!

$$= \int \left(x - 4 + \frac{6}{x + 1}\right)dx$$
$$= \frac{x^2}{2} - 4x + 6\ln|x + 1| + C$$

2d) $\int_0^2 \frac{x^2 - 2}{x + 1}\,dx$

2e) $\int \frac{x^2 - 4}{x}\,dx$

3a) $\int \frac{1}{x \ln x^3}\,dx$

$= \int \frac{1}{x \bullet 3\ln x}\,dx = \frac{1}{3}\int \frac{1}{x \ln x}\,dx$

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

So... $\frac{1}{3}\int \frac{1}{x \bullet u} \bullet x\,du = \frac{1}{3}\int \frac{1}{u}\,du$

$= \frac{1}{3}\ln|u| + C$

$= \frac{1}{3}\ln|\ln x| + C$

3b) $\int \frac{(\ln x)^2}{x}\,dx$

3c) $\int_1^e \frac{(1 + \ln x)^2}{x}\,dx$

4a) $\int \csc 2x\,dx$

$\int \csc(2x)\,dx$

Let $u = 2x$

$\frac{du}{dx} = 2$

So... $\frac{1}{2}\csc(u)\,du$

$= -\frac{1}{2}\ln|\csc(u) + \cot(u)| + C$

$= -\frac{1}{2}\ln|\csc(2x) + \cot(2x)| + C$

4b) $\int (\sec 2x + \tan 2x)\,dx$

4c) $\int \cot\frac{\theta}{3}\,d\theta$

Solve the differential equation with the given solution.

5a) $\frac{dy}{dx} = \frac{3}{2 - x}, \quad (1, 0)$

$$y = \int \frac{3}{2 - x}\, dx$$

$$= -3\int \frac{1}{x - 2}\, dx$$

$$= -3 \ln|x - 2| + C$$

$$(1, 0): \ 0 = -3 \ln|1 - 2| + C \Rightarrow C = 0$$

$$y = -3 \ln|x - 2|$$

5b) $\frac{dy}{dx} = \frac{1}{x + 2}, \quad (0, 1)$

5c) $\frac{ds}{d\theta} = \tan 2\theta, \quad (0, 2)$

5d) $\frac{dy}{dx} = 1 + \frac{1}{x}, \quad (1, 4)$

6) AP MULTIPLE CHOICE EXAMPLES

1) Let $F(x)$ be an antiderivative of $\frac{(\ln x)^3}{x}$. If $F(1) = 0$, then $F(9) =$

(A) 0.048 (B) 0.144 (C) 5.827 (D) 23.308 (E) 1,640.250

2) If $\frac{dy}{dx} = \tan x$, then $y =$

(A) $\frac{1}{2}\tan^2 x + C$ (B) $\sec^2 x + C$ (C) $\ln|\sec x| + C$

(D) $\ln|\cos x| + C$ (E) $\sec x \tan x + C$

3) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin x} dx =$

(A) $\ln\sqrt{2}$ (B) $\ln\frac{\pi}{4}$ (C) $\ln\sqrt{3}$ (D) $\ln\frac{\sqrt{3}}{2}$ (E) $\ln e$

4) $\int_1^2 \frac{x-4}{x^2} dx =$

(A) $-\frac{1}{2}$ (B) $\ln 2 - 2$ (C) $\ln 2$ (D) 2 (E) $\ln 2 + 2$

5) If $\frac{dy}{dx} = 4y$ and if $y = 4$ when $x = 0$, then $y =$

(A) $4e^{4x}$ (B) e^{4x} (C) $3 + e^{4x}$ (D) $4 + e^{4x}$ (E) $2x^2 + 4$

6) $\int_1^2 \frac{x+1}{x^2+2x}dx =$

(A) $\ln 8 - \ln 3$ (B) $\frac{\ln 8 - \ln 3}{2}$ (C) $\ln 8$ (D) $\frac{3\ln 2}{2}$ (E) $\frac{3\ln 2 + 2}{2}$

7) At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

8) If $f(x) = \begin{cases} x & \text{for } x \le 1 \\ \frac{1}{x} & \text{for } x > 1, \end{cases}$ then $\int_0^e f(x)dx =$

(A) 0 (B) $\frac{3}{2}$ (C) 2 (D) e (E) $e + \frac{1}{2}$