Optimization

w-up: Express the distance between (0, 2) and (x, y) IN TERMS OF "x" if (x, y) is a coordinate on $y=-2x+5$.

To find the **maximum/minimum** for **any function**, we apply the first derivative test and determine if a sign change occurs OR apply the second derivative test to see if value(that creates slope of zero) occurs on a CCUP/CCDOWN interval.

To solve optimization applications problems

1) Write a formula related to the problem so that the DEPENDENT(traditionally the y-value) variable is the quantity to be minimized/maximized.

2) Reduce the equation to having a single independent variable(traditionally the x-value) which sometimes involves using a secondary equation and substitution.

3) Differentiate the equation and set it equal to zero. Solve the equation and use these critical values to verify that it occurs at a min/max a formula related to the problem so that the DEPENDENT variable is the quantity to be minimized/maximized.

NOTE: Keywords to notice when a real function is to be optimized are:

LARGEST, SMALLEST, CLOSEST, FURTHEST, MOST, LEAST, GREATEST, ETC…

EXAMPLES

These examples will put to test if you are "an expert in algebra"

DISTANCE ON THE COORDINATE PLANE

1) The point on the curve $2y = x^2$ nearest to $(4,1)$ is

(A) $(0,0)$ (B) $(2,2)$ (C) $\left(\sqrt{2},1\right)$ (D) $\left(2\sqrt{2},4\right)$ (E) $(4,8)$

AREA/VOLUME OF BASIC GEOMETRIC FIGURES

2) What is the area of the largest rectangle that can be inscribed in the ellipse $4x^2 + 9y^2 = 36$?

(A) $6\sqrt{2}$ (B) 12 (C) 24 (D) $24\sqrt{2}$ (E) 36

3) Consider all right circular cylinders for which the sum of the height and circumference is 30 centimeters. What is the radius of the one with maximum volume?

(A) 3 cm (B) 10 cm (C) 20 cm (D) $\frac{30}{\pi^2}$ cm (E) $\frac{10}{\pi}$ cm