CHAPTER 4
Integration

Section 4.1 Antiderivatives and Indefinite Integration

Solutions to Exercises

1. $\dfrac{d}{dx}\left(\dfrac{3}{x^3}+C\right)=\dfrac{d}{dx}(3x^{-3}+C)=-9x^{-4}=\dfrac{-9}{x^4}$

2. $\dfrac{d}{dx}\left(x^4+\dfrac{1}{x}+C\right)=4x^3-\dfrac{1}{x^2}$

3. $\dfrac{d}{dx}\left(\dfrac{1}{3}x^3-4x+C\right)=x^2-4=(x-2)(x+2)$

4. $\dfrac{d}{dx}\left(\dfrac{2(x^2+3)}{3\sqrt{x}}+C\right)=\dfrac{d}{dx}\left(\dfrac{2}{3}x^{3/2}+2x^{-1/2}+C\right)$

$=x^{1/2}-x^{-3/2}=\dfrac{x^2-1}{x^{3/2}}$

	Given	*Rewrite*	*Integrate*	*Simplify*
5.	$\int\sqrt[3]{x}\,dx$	$\int x^{1/3}\,dx$	$\dfrac{x^{4/3}}{4/3}+C$	$\dfrac{3}{4}x^{4/3}+C$
6.	$\int\dfrac{1}{x^2}\,dx$	$\int x^{-2}\,dx$	$\dfrac{x^{-1}}{-1}+C$	$-\dfrac{1}{x}+C$
7.	$\int\dfrac{1}{x\sqrt{x}}\,dx$	$\int x^{-3/2}\,dx$	$\dfrac{x^{-1/2}}{-1/2}+C$	$-\dfrac{2}{\sqrt{x}}+C$
8.	$\int x(x^2+3)\,dx$	$\int(x^3+3x)\,dx$	$\dfrac{x^4}{4}+3\left(\dfrac{x^2}{2}\right)+C$	$\dfrac{1}{4}x^4+\dfrac{3}{2}x^2+C$
9.	$\int\dfrac{1}{2x^3}\,dx$	$\dfrac{1}{2}\int x^{-3}\,dx$	$\dfrac{1}{2}\left(\dfrac{x^{-2}}{-2}\right)+C$	$-\dfrac{1}{4x^2}+C$
10.	$\int\dfrac{1}{(2x)^3}\,dx$	$\dfrac{1}{8}\int x^{-3}\,dx$	$\dfrac{1}{8}\left(\dfrac{x^{-2}}{-2}\right)+C$	$-\dfrac{1}{16x^2}+C$

11. $\dfrac{dy}{dt}=3t^2$

$y=t^3+C$

Check: $\dfrac{d}{dt}[t^3+C]=3t^2$

12. $\dfrac{dr}{d\theta}=\pi$

$r=\pi\theta+C$

Check: $\dfrac{d}{d\theta}[\pi\theta+C]=\pi$

13. $\dfrac{dy}{dx}=x^{3/2}$

$y=\dfrac{2}{5}x^{5/2}+C$

Check: $\dfrac{d}{dx}\left[\dfrac{2}{5}x^{5/2}+C\right]=x^{3/2}$

14. $\dfrac{dy}{dx}=3x^{-4}$

$y=-x^{-3}+C=-\dfrac{1}{x^3}+C$

Check: $\dfrac{d}{dx}[-x^{-3}+C]=3x^{-4}$

15. $\displaystyle\int(x^3+2)\,dx=\frac{1}{4}x^4+2x+C$

Check: $\dfrac{d}{dx}\left(\dfrac{1}{4}x^4+2x+C\right)=x^3+2$

16. $\int (x^2 - 2x + 3)\,dx = \frac{1}{3}x^3 - x^2 + 3x + C$

Check: $\frac{d}{dx}\left(\frac{1}{3}x^3 - x^2 + 3x + C\right) = x^2 - 2x + 3$

17. $\int (x^{3/2} + 2x + 1)\,dx = \frac{2}{5}x^{5/2} + x^2 + x + C$

Check: $\frac{d}{dx}\left(\frac{2}{5}x^{5/2} + x^2 + x + C\right) = x^{3/2} + 2x + 1$

18. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx = \int \left(x^{1/2} + \frac{1}{2}x^{-1/2}\right) dx = \frac{x^{3/2}}{3/2} + \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + C = \frac{2}{3}x^{3/2} + x^{1/2} + C$

Check: $\frac{d}{dx}\left(\frac{2}{3}x^{3/2} + x^{1/2} + C\right) = x^{1/2} + \frac{1}{2}x^{-1/2} = \sqrt{x} + \frac{1}{2\sqrt{x}}$

19. $\int \sqrt[3]{x^2}\,dx = \int x^{2/3}\,dx = \frac{x^{5/3}}{5/3} + C = \frac{3}{5}x^{5/3} + C$

Check: $\frac{d}{dx}\left(\frac{3}{5}x^{5/3} + C\right) = x^{2/3} = \sqrt[3]{x^2}$

20. $\int \left(\sqrt[4]{x^3} + 1\right) dx = \int (x^{3/4} + 1)\,dx = \frac{4}{7}x^{7/4} + x + C$

Check: $\frac{d}{dx}\left(\frac{4}{7}x^{7/4} + x + C\right) = x^{3/4} + 1 = \sqrt[4]{x^3} + 1$

21. $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C = -\frac{1}{2x^2} + C$

Check: $\frac{d}{dx}\left(-\frac{1}{2x^2} + C\right) = \frac{1}{x^3}$

22. $\int \frac{1}{x^4}\,dx = \int x^{-4}\,dx = \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} + C$

Check: $\frac{d}{dx}\left(-\frac{1}{3x^3} + C\right) = \frac{1}{x^4}$

23. $\int \frac{x^2 + x + 1}{\sqrt{x}}\,dx = \int (x^{3/2} + x^{1/2} + x^{-1/2})\,dx = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C = \frac{2}{15}x^{1/2}(3x^2 + 5x + 15) + C$

Check: $\frac{d}{dx}\left(\frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + 2x^{1/2} + C\right) = x^{3/2} + x^{1/2} + x^{-1/2} = \frac{x^2 + x + 1}{\sqrt{x}}$

24. $\int \frac{x^2 + 1}{x^2}\,dx = \int (1 + x^{-2})\,dx$

$= x + \frac{x^{-1}}{-1} + C = x - \frac{1}{x} + C$

Check: $\frac{d}{dx}\left(x - \frac{1}{x} + C\right) = 1 + \frac{1}{x^2} = \frac{x^2 + 1}{x^2}$

25. $\int (x + 1)(3x - 2)\,dx = \int (3x^2 + x - 2)\,dx$

$= x^3 + \frac{1}{2}x^2 - 2x + C$

Check: $\frac{d}{dx}\left(x^3 + \frac{1}{2}x^2 - 2x + C\right) = 3x^2 + x - 2$

$= (x + 1)(3x - 2)$

26. $\int (2t^2 - 1)^2\,dt = \int (4t^4 - 4t^2 + 1)\,dt$

$= \frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C$

Check: $\frac{d}{dt}\left(\frac{4}{5}t^5 - \frac{4}{3}t^3 + t + C\right) = 4t^4 - 4t^2 + 1$

$= (2t^2 - 1)^2$

27. $\int y^2\sqrt{y}\,dy = \int y^{5/2}\,dy = \frac{2}{7}y^{7/2} + C$

Check: $\frac{d}{dy}\left(\frac{2}{7}y^{7/2} + C\right) = y^{5/2} = y^2\sqrt{y}$

28. $\int (1 + 3t)t^2\,dt = \int (t^2 + 3t^3)\,dt = \frac{1}{3}t^3 + \frac{3}{4}t^4 + C$

Check: $\frac{d}{dt}\left(\frac{1}{3}t^3 + \frac{3}{4}t^4 + C\right) = t^2 + 3t^3 = (1 + 3t)t^2$

29. $\int dx = \int 1\,dx = x + C$

Check: $\frac{d}{dx}(x + C) = 1$

30. $\int 3\,dt = 3t + C$

Check: $\dfrac{d}{dt}(3t + C) = 3$

31. $\int (2\sin x + 3\cos x)\,dx = -2\cos x + 3\sin x + C$

Check: $\dfrac{d}{dx}(-2\cos x + 3\sin x + C) = 2\sin x + 3\cos x$

32. $\int (t^2 - \sin t)\,dt = \dfrac{1}{3}t^3 + \cos t + C$

Check: $\dfrac{d}{dt}\left(\dfrac{1}{3}t^3 + \cos t + C\right) = t^2 - \sin t$

33. $\int (1 - \csc t \cot t)\,dt = t + \csc t + C$

Check: $\dfrac{d}{dt}(t + \csc t + C) = 1 - \csc t \cot t$

34. $\int (\theta^2 + \sec^2\theta)\,d\theta = \dfrac{1}{3}\theta^3 + \tan\theta + C$

Check: $\dfrac{d}{d\theta}\left(\dfrac{1}{3}\theta^3 + \tan\theta + C\right) = \theta^2 + \sec^2\theta$

35. $\int (\sec^2\theta - \sin\theta)\,d\theta = \tan\theta + \cos\theta + C$

Check: $\dfrac{d}{d\theta}(\tan\theta + \cos\theta + C) = \sec^2\theta - \sin\theta$

36. $\int \sec y(\tan y - \sec y)\,dy = \int (\sec y \tan y - \sec^2 y)\,dy$

$= \sec y - \tan y + C$

Check: $\dfrac{d}{dy}(\sec y - \tan y + C) = \sec y \tan y - \sec^2 y$

$= \sec y(\tan y - \sec y)$

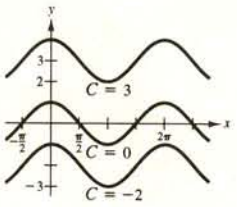
37. $\int (\tan^2 y + 1)\,dy = \int \sec^2 y\,dy = \tan y + C$

Check: $\dfrac{d}{dy}(\tan y + C) = \sec^2 y = \tan^2 y + 1$

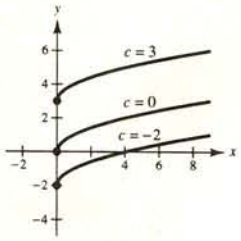
38. $\displaystyle\int \frac{\sin x}{1 - \sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos}\right)\left(\frac{\sin x}{\cos x}\right)dx = \int \sec x \tan x\,dx = \sec x + C$

Check: $\dfrac{d}{dx}(\sec x + C) = \sec x \tan x = \dfrac{\sin x}{\cos^2 x} = \dfrac{\sin x}{1 - \sin^2 x}$

39. $f(x) = \cos x$

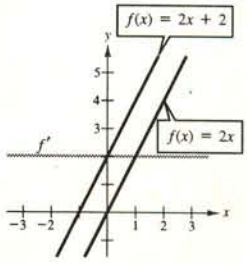


40. $f(x) = \sqrt{x}$



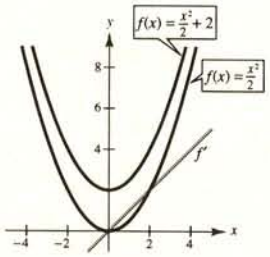
41. $f'(x) = 2$

$f(x) = 2x + C$



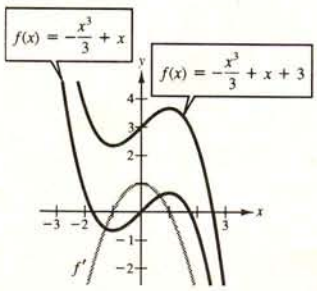
42. $f'(x) = x$

$f(x) = \dfrac{x^2}{2} + C$



43. $f'(x) = 1 - x^2$

$f(x) = x - \dfrac{x^3}{3} + C$



44. $f'(x) = \dfrac{1}{x^2}$

$f(x) = -\dfrac{1}{x} + C$