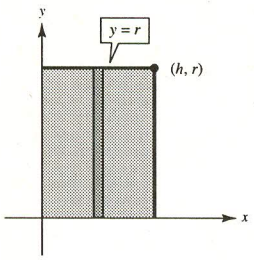
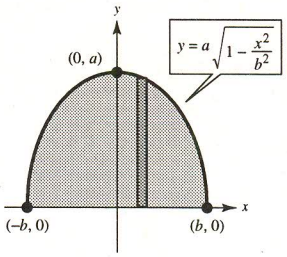
53. (a) $\pi\int_0^h r^2\,dx$ (ii)

is the volume of a right circular cylinder with radius r and height h.



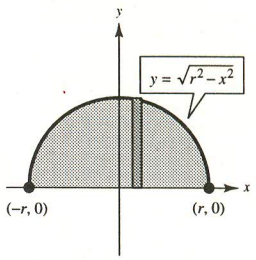
(b) $\pi\int_{-b}^{b}\left(a\sqrt{1-\frac{x^2}{b^2}}\right)^2 dx$ (iv)

is the volume of an ellipsoid with axes $2a$ and $2b$.



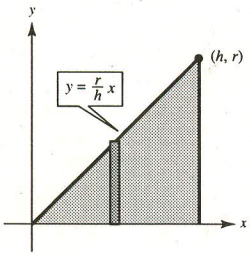
(c) $\pi\int_{-r}^{r}\left(\sqrt{r^2-x^2}\right)^2 dx$ (iii)

is the volume of a sphere with radius r.



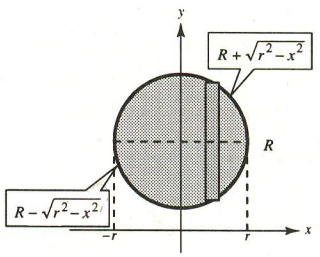
(d) $\pi\int_0^h\left(\frac{rx}{h}\right)^2 dx$ (i)

is the volume of a right circular cone with the radius of the base as r and height h.



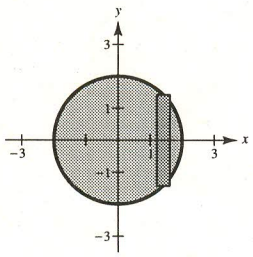
(e) $\pi\int_{-r}^{r}\left[\left(R+\sqrt{r^2-x^2}\right)^2-\left(R-\sqrt{r^2-x^2}\right)^2\right]dx$ (v)

is the volume of a torus with the radius of its circular cross section as r and the distance from the axis of the torus to the center of its cross section as R.



54. $V = \frac{1}{2}(8)(1)(2) = 8\ \text{m}^3$

55.

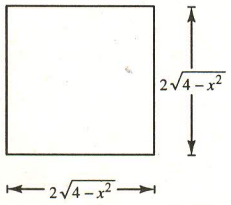


Base of Cross Section $= 2\sqrt{4-x^2}$

(a) $A(x) = b^2 = \left(2\sqrt{4-x^2}\right)^2$

$$V = \int_{-2}^{2} 4(4-x^2)\,dx$$

$$= 4\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{128}{3}$$

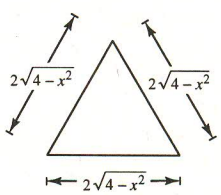


(b) $A(x) = \frac{1}{2}bh = \frac{1}{2}\left(2\sqrt{4-x^2}\right)\left(\sqrt{3}\sqrt{4-x^2}\right)$

$$= \sqrt{3}\,(4-x^2)$$

$$V = \sqrt{3}\int_{-2}^{2}(4-x^2)\,dx$$

$$= \sqrt{3}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32\sqrt{3}}{3}$$



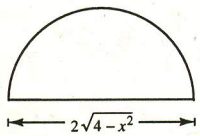
—CONTINUED—

55. —CONTINUED—

(c) $A(x) = \frac{1}{2}\pi r^2$

$= \frac{\pi}{2}\left(\sqrt{4 - x^2}\right)^2 = \frac{\pi}{2}(4 - x^2)$

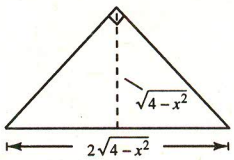
$V = \frac{\pi}{2}\int_{-2}^{2}(4 - x^2)\,dx = \frac{\pi}{2}\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{16\pi}{3}$



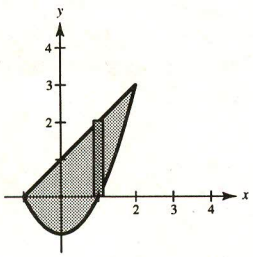
(d) $A(x) = \frac{1}{2}bh$

$= \frac{1}{2}\left(2\sqrt{4 - x^2}\right)\left(\sqrt{4 - x^2}\right) = 4 - x^2$

$V = \int_{-2}^{2}(4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \frac{32}{3}$



56.



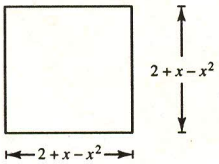
Base of Cross Section $= (x + 1) - (x^2 - 1) = 2 + x - x^2$

(a) $A(x) = b^2 = (2 + x - x^2)^2$

$= 4 + 4x - 3x^2 - 2x^3 + x^4$

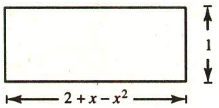
$V = \int_{-1}^{2}(4 + 4x - 3x^2 - 2x^3 + x^4)\,dx$

$= \left[4x + 2x^3 - x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right]_{-1}^{2} = \frac{81}{10}$

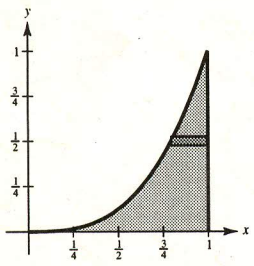


(b) $A(x) = bh = (2 + x - x^2)1$

$V = \int_{-1}^{2}(2 + x - x^2)\,dx = \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2} = \frac{9}{2}$



57.



Base of Cross Section $= 1 - \sqrt[3]{y}$

—CONTINUED—

(a) $A(y) = b^2 = \left(1 - \sqrt[3]{y}\right)^2$

$V = \int_0^1 \left(1 - \sqrt[3]{y}\right)^2 dy$

$= \int_0^1 (1 - 2y^{1/3} + y^{2/3})\,dy$

$= \left[y - \frac{3}{2}y^{4/3} + \frac{3}{5}y^{5/3}\right]_0^1 = \frac{1}{10}$