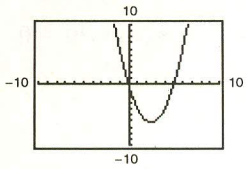
Section 1.3 Evaluating Limits Analytically

1.

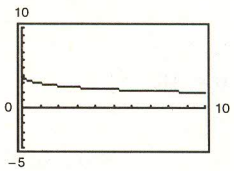


$h(x) = x^2 - 5x$

(a) $\lim_{x\to 5} h(x) = 0$

(b) $\lim_{x\to -1} h(x) = 6$

2.

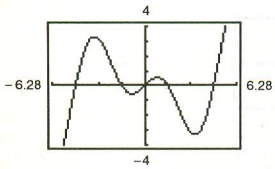


$g(x) = \dfrac{12(\sqrt{x} - 3)}{x - 9}$

(a) $\lim_{x\to 4} g(x) = 2.4$

(b) $\lim_{x\to 0} g(x) = 4$

3.



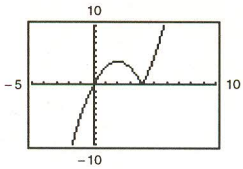
$f(x) = x \cos x$

(a) $\lim_{x\to 0} f(x) = 0$

(b) $\lim_{x\to \pi/3} f(x) \approx 0.524$

$\left(= \dfrac{\pi}{6}\right)$

4.



$f(t) = t|t - 4|$

(a) $\lim_{t\to 4} f(t) = 0$

(b) $\lim_{t\to -1} f(t) = -5$

5. $\lim_{x\to 4} x^2 = 4^2 = 16$

6. $\lim_{x\to -3} (3x + 2) = 3(-3) + 2 = 7$

7. $\lim_{x\to 0} (2x - 1) = 2(0) - 1 = -1$

8. $\lim_{x\to 1} (-x^2 + 1) = -(1)^2 + 1 = 0$

9. $\lim_{x\to 2} (-x^2 + x - 2) = -(2)^2 + (2) - 2 = -4$

10. $\lim_{x\to 1} (3x^3 - 2x^2 + 4) = 3(1)^3 - 2(1)^2 + 4 = 5$

11. $\lim_{x\to 3} \sqrt{x + 1} = \sqrt{3 + 1} = 2$

12. $\lim_{x\to 4} \sqrt[3]{x + 4} = \sqrt[3]{4 + 4} = 2$

13. $\lim_{x\to -4} (x + 3)^2 = (-4 + 3)^2 = 1$

14. $\lim_{x\to 0} (2x - 1)^3 = [2(0) - 1]^3 = -1$

15. $\lim_{x\to 2} \dfrac{1}{x} = \dfrac{1}{2}$

16. $\lim_{x\to -3} \dfrac{2}{x + 2} = \dfrac{2}{-3 + 2} = -2$

17. $\lim_{x\to -1} \dfrac{x^2 + 1}{x} = \dfrac{(-1)^2 + 1}{-1} = -2$

18. $\lim_{x\to 3} \dfrac{\sqrt{x + 1}}{x - 4} = \dfrac{\sqrt{3 + 1}}{3 - 4} = -2$

19. $\lim_{x\to \pi/2} \sin x = \sin \dfrac{\pi}{2} = 1$

20. $\lim_{x\to \pi} \tan x = \tan \pi = 0$

21. $\lim_{x\to 1} \cos \pi x = \cos \pi = -1$

22. $\lim_{x\to 1} \sin \dfrac{\pi x}{2} = \sin \dfrac{\pi}{2} = 1$

23. $\lim_{x\to 0} \sec 2x = \sec 0 = 1$

24. $\lim_{x\to \pi} \cos 3x = \cos 3\pi = -1$

25. $\lim_{x\to 5\pi/6} \sin x = \sin \dfrac{5\pi}{6} = \dfrac{1}{2}$

26. $\lim_{x\to 5\pi/3} \cos x = \cos \dfrac{5\pi}{3} = \dfrac{1}{2}$

27. $\lim_{x\to 3} \tan\left(\dfrac{\pi x}{4}\right) = \tan \dfrac{3\pi}{4} = -1$

28. $\lim_{x\to 7} \sec\left(\dfrac{\pi x}{6}\right) = \sec \dfrac{7\pi}{6} = \dfrac{-2\sqrt{3}}{3}$

29. (a) $\lim_{x\to c}[5g(x)] = 5\lim_{x\to c} g(x) = 5(3) = 15$

(b) $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = 2 + 3 = 5$

(c) $\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = (2)(3) = 6$

(d) $\lim_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \dfrac{2}{3}$

30. (a) $\lim_{x\to c}[4f(x)] = 4\lim_{x\to c} f(x) = 4\left(\dfrac{3}{2}\right) = 6$

(b) $\lim_{x\to c}[f(x) + g(x)] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x) = \dfrac{3}{2} + \dfrac{1}{2} = 2$

(c) $\lim_{x\to c}[f(x)g(x)] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right] = \left(\dfrac{3}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{3}{4}$

(d) $\lim_{x\to c}\dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \dfrac{3/2}{1/2} = 3$

31. (a) $\lim_{x\to c}[f(x)]^3 = \left[\lim_{x\to c} f(x)\right]^3 = (4)^3 = 64$

(b) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{\lim_{x\to c} f(x)} = \sqrt{4} = 2$

(c) $\lim_{x\to c}[3f(x)] = 3\lim_{x\to c} f(x) = 3(4) = 12$

(d) $\lim_{x\to c}[f(x)]^{3/2} = \left[\lim_{x\to c} f(x)\right]^{3/2} = (4)^{3/2} = 8$

32. (a) $\lim_{x\to c}\sqrt[3]{f(x)} = \sqrt[3]{\lim_{x\to c} f(x)} = \sqrt[3]{27} = 3$

(b) $\lim_{x\to c}\dfrac{f(x)}{18} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} 18} = \dfrac{27}{18} = \dfrac{3}{2}$

(c) $\lim_{x\to c}[f(x)]^2 = \left[\lim_{x\to c} f(x)\right]^2 = (27)^2 = 729$

(d) $\lim_{x\to c}[f(x)]^{2/3} = \left[\lim_{x\to c} f(x)\right]^{2/3} = (27)^{2/3} = 9$

33. $f(x) = -2x + 1$ and $g(x) = \dfrac{-2x^2 + x}{x}$ agree except at $x = 0$.

(a) $\lim_{x\to 0} g(x) = \lim_{x\to 0} f(x) = 1$

(b) $\lim_{x\to -1} g(x) = \lim_{x\to -1} f(x) = 3$

34. $f(x) = x - 3$ and $h(x) = \dfrac{x^2 - 3x}{x}$ agree except at $x = 0$.

(a) $\lim_{x\to -2} h(x) = \lim_{x\to -2} f(x) = -5$

(b) $\lim_{x\to 0} h(x) = \lim_{x\to 0} f(x) = -3$

35. $f(x) = x(x + 1)$ and $g(x) = \dfrac{x^3 - x}{x - 1}$ agree except at $x = 1$.

(a) $\lim_{x\to 1} g(x) = \lim_{x\to 1} f(x) = 2$

(b) $\lim_{x\to -1} g(x) = \lim_{x\to -1} f(x) = 0$

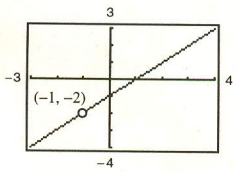
36. $g(x) = \dfrac{1}{x - 1}$ and $f(x) = \dfrac{x}{x^2 - x}$ agree except at $x = 0$.

(a) $\lim_{x\to 1} f(x)$ does not exist.

(b) $\lim_{x\to 0} f(x) = -1$

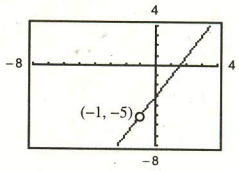
37. $f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.

$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = -2$



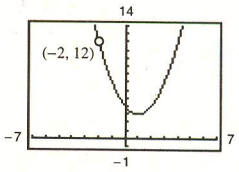
38. $f(x) = \dfrac{2x^2 - x - 3}{x + 1}$ and $g(x) = 2x - 3$ agree except at $x = -1$.

$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = -5$



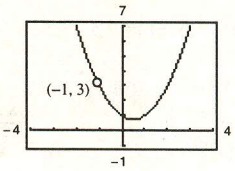
39. $f(x) = \dfrac{x^3 + 8}{x + 2}$ and $g(x) = x^2 - 2x + 4$ agree except at $x = -2$.

$\lim_{x\to -2} f(x) = \lim_{x\to -2} g(x) = 12$



40. $f(x) = \dfrac{x^3 + 1}{x + 1}$ and $g(x) = x^2 - x + 1$ agree except at $x = -1$.

$$\lim_{x \to -1} f(x) = \lim_{x \to -1} g(x) = 3$$



41. $$\lim_{x \to 5} \frac{x - 5}{x^2 - 25} = \lim_{x \to 5} \frac{x - 5}{(x + 5)(x - 5)} = \lim_{x \to 5} \frac{1}{x + 5} = \frac{1}{10}$$

42. $$\lim_{x \to 2} \frac{2 - x}{x^2 - 4} = \lim_{x \to 2} \frac{-(x - 2)}{(x - 2)(x + 2)} = \lim_{x \to 2} \frac{-1}{x + 2} = -\frac{1}{4}$$

43. $$\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - 1} = \lim_{x \to 1} \frac{(x + 2)(x - 1)}{(x + 1)(x - 1)} = \lim_{x \to 1} \frac{x + 2}{x + 1} = \frac{3}{2}$$

44. $$\lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x} = \lim_{x \to 0} \frac{\sqrt{2 + x} - \sqrt{2}}{x} \cdot \frac{\sqrt{2 + x} + \sqrt{2}}{\sqrt{2 + x} + \sqrt{2}}$$
$$= \lim_{x \to 0} \frac{2 + x - 2}{\left(\sqrt{2 + x} + \sqrt{2}\right)x} = \lim_{x \to 0} \frac{1}{\sqrt{2 + x} + \sqrt{2}} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$$

45. $$\lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x} = \lim_{x \to 0} \frac{\sqrt{3 + x} - \sqrt{3}}{x} \cdot \frac{\sqrt{3 + x} + \sqrt{3}}{\sqrt{3 + x} + \sqrt{3}}$$
$$= \lim_{x \to 0} \frac{3 + x - 3}{\left(\sqrt{3 + x} + \sqrt{3}\right)x} = \lim_{x \to 0} \frac{1}{\left(\sqrt{3 + x} + \sqrt{3}\right)} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

46. $$\lim_{x \to 0} \frac{\dfrac{1}{x + 4} - \dfrac{1}{4}}{x} = \lim_{x \to 0} \frac{\dfrac{4 - (x + 4)}{4(x + 4)}}{x} = \lim_{x \to 0} \frac{-1}{4(x + 4)} = -\frac{1}{16}$$

47. $$\lim_{x \to 0} \frac{\dfrac{1}{2 + x} - \dfrac{1}{2}}{x} = \lim_{x \to 0} \frac{\dfrac{2 - (2 + x)}{2(2 + x)}}{x} = \lim_{x \to 0} \frac{-1}{2(2 + x)} = -\frac{1}{4}$$

48. $$\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} = \lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3} \cdot \frac{\sqrt{x + 1} + 2}{\sqrt{x + 1} + 2} = \lim_{x \to 3} \frac{x - 3}{(x - 3)\left[\sqrt{x + 1} + 2\right]} = \lim_{x \to 3} \frac{1}{\sqrt{x + 1} + 2} = \frac{1}{4}$$

49. $$\lim_{\Delta x \to 0} \frac{2(x + \Delta x) - 2x}{\Delta x} = \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 2x}{\Delta x} = \lim_{\Delta x \to 0} 2 = 2$$

50. $$\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x)}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

51. $$\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - 2(x + \Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x - 2) = 2x - 2$$