100. Define $f(x) = f_2(x) - f_1(x)$. Since f_1 and f_2 are continuous on $[a, b]$, so is f.

$$f(a) = f_2(a) - f_1(a) > 0 \quad \text{and} \quad f(b) = f_2(b) - f_1(b) < 0.$$

By the Intermediate Value Theorem, there exists c in $[a, b]$ such that $f(c) = 0$.

$$f(c) = f_2(c) - f_1(c) = 0 \Longrightarrow f_1(c) = f_2(c)$$

Section 1.5 Infinite Limits

1. $\lim_{x \to -2^+} \frac{1}{(x+2)^2} = \infty$

$\lim_{x \to -2^-} \frac{1}{(x+2)^2} = \infty$

2. $\lim_{x \to -2^+} \frac{1}{x+2} = \infty$

$\lim_{x \to -2^-} \frac{1}{x+2} = -\infty$

3. $\lim_{x \to -2^+} \tan \frac{\pi x}{4} = -\infty$

$\lim_{x \to -2^-} \tan \frac{\pi x}{4} = \infty$

4. $\lim_{x \to -2^+} \sec \frac{\pi x}{4} = \infty$

$\lim_{x \to -2^-} \sec \frac{\pi x}{4} = -\infty$

5. $f(x) = \frac{1}{x^2 - 9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	0.308	1.639	16.64	166.6	-166.7	-16.69	-1.695	-0.364

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

6. $f(x) = \frac{x}{x^2 - 9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	-1.077	-5.082	-50.08	-500.1	499.9	49.92	4.915	0.9091

$\lim_{x \to -3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = \infty$

7. $f(x) = \frac{x^2}{x^2 - 9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	3.769	15.75	150.8	1501	-1499	-149.3	-14.25	-2.273

$\lim_{x \to -3^-} f(x) = \infty$

$\lim_{x \to -3^+} f(x) = -\infty$

8. $f(x) = \sec \dfrac{\pi x}{6}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	-3.864	-19.11	-191.0	-1910	1910	191.0	19.11	3.864

$$\lim_{x\to -3^-} f(x) = -\infty$$

$$\lim_{x\to -3^+} f(x) = \infty$$

9. $\displaystyle\lim_{x\to 0^+} \frac{1}{x^2} = \infty = \lim_{x\to 0^-} \frac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

10. $\displaystyle\lim_{x\to 2^+} \frac{4}{(x-2)^3} = \infty$

$$\lim_{x\to 2^-} \frac{4}{(x-2)^3} = -\infty$$

Therefore, $x = 2$ is a vertical asymptote.

11. $\displaystyle\lim_{x\to 2^+} \frac{x^2-2}{(x-2)(x+1)} = \infty$

$$\lim_{x\to 2^-} \frac{x^2-2}{(x-2)(x+1)} = -\infty$$

Therefore, $x = 2$ is a vertical asymptote.

$$\lim_{x\to -1^+} \frac{x^2-2}{(x-2)(x+1)} = \infty$$

$$\lim_{x\to -1^-} \frac{x^2-2}{(x-2)(x+1)} = -\infty$$

Therefore, $x = -1$ is a vertical asymptote.

12. $\displaystyle\lim_{x\to 1^+} \frac{2+x}{1-x} = -\infty$

$$\lim_{x\to 1^-} \frac{2+x}{1-x} = \infty$$

Therefore, $x = 1$ is a vertical asymptote.

13. $\displaystyle\lim_{x\to -1^+} \frac{x^3}{x^2-1} = \infty$

$$\lim_{x\to -1^-} \frac{x^3}{x^2-1} = -\infty$$

Therefore, $x = -1$ is a vertical asymptote.

$$\lim_{x\to 1^+} \frac{x^3}{x^2-1} = \infty$$

$$\lim_{x\to 1^-} \frac{x^3}{x^2-1} = -\infty$$

Therefore, $x = 1$ is a vertical asymptote.

14. No vertical asymptote since the denominator is never zero.

15. $f(x) = \tan 2x = \dfrac{\sin 2x}{\cos 2x}$ has vertical asymptotes at

$$x = \frac{(2n+1)\pi}{4} = \frac{\pi}{4} + \frac{n\pi}{2},\ n \text{ any integer.}$$

16. $f(x) = \sec \pi x = \dfrac{1}{\cos \pi x}$ has vertical asymptotes at

$$x = \frac{2n+1}{2},\ n \text{ any integer.}$$

17. $\displaystyle\lim_{t\to 0^+} \left(1 - \frac{4}{t^2}\right) = -\infty = \lim_{t\to 0^-} \left(1 - \frac{4}{t^2}\right)$

Therefore, $t = 0$ is a vertical asymptote.

18. $\displaystyle\lim_{s\to 2^+} \frac{-2}{(s-2)^2} = -\infty = \lim_{s\to 2^-} \frac{-2}{(s-2)^2}$

Therefore, $s = 2$ is a vertical asymptote.

19. $\lim_{x\to -2^+} \dfrac{x}{(x+2)(x-1)} = \infty$

$\lim_{x\to -2^-} \dfrac{x}{(x+2)(x-1)} = -\infty$

Therefore, $x = -2$ is a vertical asymptote.

$\lim_{x\to 1^+} \dfrac{x}{(x+2)(x-1)} = \infty$

$\lim_{x\to 1^-} \dfrac{x}{(x+2)(x-1)} = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

20. $\lim_{x\to -3^+} \dfrac{1}{(x+3)^4} = \infty = \lim_{x\to -3^-} \dfrac{1}{(x+3)^4}$

Therefore, $x = -3$ is a vertical asymptote.

21. $f(x) = \dfrac{x^3+1}{x+1} = \dfrac{(x+1)(x^2-x+1)}{x+1}$

has no vertical asymptote since

$\lim_{x\to -1} f(x) = \lim_{x\to -1} (x^2 - x + 1) = 3,$

not infinity.

22. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2} = \dfrac{(x+2)(x-2)}{(x+2)(x^2+1)}$

has no vertical asymptote since

$\lim_{x\to -2} h(x) = \lim_{x\to -2} \dfrac{x-2}{x^2+1} = -\dfrac{4}{5},$

not infinity.

23. $s(t) = \dfrac{t}{\sin t}$ has vertical asymptotes at $t = n\pi$, n a nonzero integer. There is no vertical asymptote at $t = 0$ since

$\lim_{t\to 0} \dfrac{t}{\sin t} = 1.$

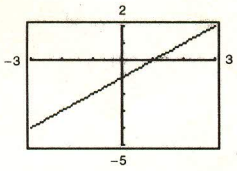
24. $g(\theta) = \dfrac{\tan\theta}{\theta} = \dfrac{\sin\theta}{\theta\cos\theta}$ has vertical asymptotes at

$\theta = \dfrac{(2n+1)\pi}{2} = \dfrac{\pi}{2} + n\pi$, n any integer.

There is no vertical asymptote at $\theta = 0$ since

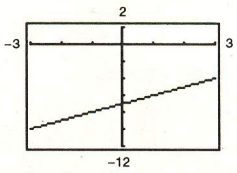
$\lim_{\theta\to 0} \dfrac{\tan\theta}{\theta} = 1.$

25. $\lim_{x\to -1} \dfrac{x^2-1}{x+1} = \lim_{x\to -1} (x-1) = -2$



Removable discontinuity at $x = -1$

26. $\lim_{x\to -1} \dfrac{x^2-6x-7}{x+1} = \lim_{x\to -1} (x-7) = -8$

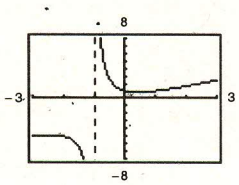


Removable discontinuity at $x = -1$

27. $\lim_{x\to -1^+} \dfrac{x^2+1}{x+1} = \infty$

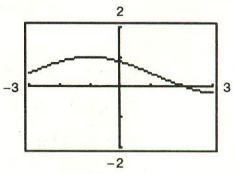
$\lim_{x\to -1^-} \dfrac{x^2+1}{x+1} = -\infty$

Vertical asymptote at $x = -1$



28. $\lim_{x\to -1} \dfrac{\sin(x+1)}{x+1} = 1$

Removable discontinuity at $x = -1$



29. $\lim_{x\to 2^+} \dfrac{x-3}{x-2} = -\infty$

30. $\lim_{x\to 1^+} \dfrac{2+x}{1-x} = -\infty$

31. $\lim_{x\to 4^+} \frac{x^2}{x^2 - 16} = \infty$

32. $\lim_{x\to 4^-} \frac{x^2}{x^2 + 16} = \frac{1}{2}$

33. $\lim_{x\to -3^-} \frac{x^2 + 2x - 3}{x^2 + x - 6} = \lim_{x\to -3^-} \frac{x - 1}{x - 2} = \frac{4}{5}$

34. $\lim_{x\to -(1/2)^+} \frac{6x^2 + x - 1}{4x^2 - 4x - 3} = \lim_{x\to -(1/2)^+} \frac{3x - 1}{2x - 3} = \frac{5}{8}$

35. $\lim_{x\to 0^-} \left(1 + \frac{1}{x}\right) = -\infty$

36. $\lim_{x\to 0^-} \left(x^2 - \frac{1}{x}\right) = \infty$

37. $\lim_{x\to 0^+} \frac{2}{\sin x} = \infty$

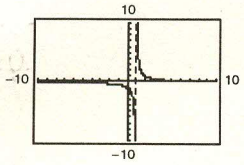
38. $\lim_{x\to (\pi/2)^+} \frac{-2}{\cos x} = \infty$

39. $\lim_{x\to 1} \frac{x^2 - x}{(x^2 + 1)(x - 1)} = \lim_{x\to 1} \frac{x}{x^2 + 1} = \frac{1}{2}$

40. $\lim_{x\to 3} \frac{x - 2}{x^2} = \frac{1}{9}$

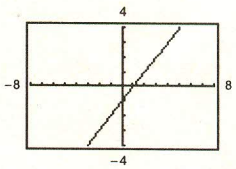
41. $f(x) = \frac{x^2 + x + 1}{x^3 - 1}$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{1}{x - 1} = \infty$



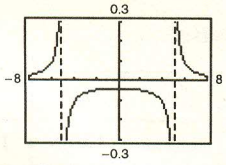
42. $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x - 1) = 0$



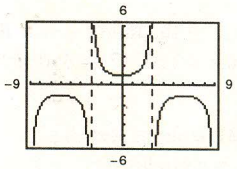
43. $f(x) = \frac{1}{x^2 - 25}$

$\lim_{x\to 5^-} f(x) = -\infty$



44. $f(x) = \sec \frac{\pi x}{6}$

$\lim_{x\to 3^+} f(x) = -\infty$



45. $S = \frac{k}{1 - r}$, $0 < |r| < 1$. Assume $k \neq 0$.

$\lim_{r\to 1^-} S = \lim_{r\to 1^-} \frac{k}{1 - r} = \infty$ (or $-\infty$ if $k < 0$)

46. $P = \frac{k}{V}$

$\lim_{V\to 0^+} \frac{k}{V} = k(\infty) = \infty$ (In this case we know that $k > 0$.)

47. (a) $r = \frac{2(7)}{\sqrt{625 - 49}} = \frac{7}{12}$ ft/sec

(b) $r = \frac{2(15)}{\sqrt{625 - 225}} = \frac{3}{2}$ ft/sec

(c) $\lim_{x\to 25^-} \frac{2x}{\sqrt{625 - x^2}} = \infty$

48. (a) $r = 50\pi \sec^2 \frac{\pi}{6} = \frac{200\pi}{3}$ ft/sec

(b) $r = 50\pi \sec^2 \frac{\pi}{3} = 200\pi$ ft/sec

(c) $\lim_{\theta\to (\pi/2)^-} [50\pi \sec^2 \theta] = \infty$