Related Rates

Differentiate $y = x^2 - 3x$ with respect to x

Remember, we can differentiate with respect to any variable. When differentiating "with respect to time" while using real world formulas results in real world Rates of Change.

So… $\frac{dA}{dt}$ could represent rate of change of area and $\frac{dV}{dt}$ could represent rate of change of volume, etc…

Differentiate $y = x^2 - 3x$ with respect to t.

Now find $\frac{dx}{dt}$ if $\frac{dy}{dt} = 5$ when $x = 1$.

Note: Be sure to use a negative value **for rates of changes** where the quantity is decreasing over time!

Related Rate Application Problems

1) Write a formula(*equation*) related to problem.

2) Differentiate only after variables match given information. If not, rewrite one variable in terms of another and substitute in first.

3) After differentiation, replace variable with given or found values and solve for missing quantity.

Basic Equation and Formula Types

EX) The positive variables p and c change with respect to time t. The relationship between p and c is given by the equation $p^2 = (20 - c)^3$. At the instant when $\frac{dp}{dt} = 41$ and $c = 15$, what is the value of $\frac{dc}{dt}$?

(A) $-\frac{82}{75}$ (B) $-\frac{2\sqrt{5}}{3}$ (C) $-\frac{3\sqrt{5}}{2}$ (D) $-\frac{82\sqrt{5}}{15}$

EX) The radius r of a sphere is increasing at the uniform rate of 0.3 inches per second. At the instant when the surface area S becomes 100π square inches, what is the rate of increase, in cubic inches per second, in the volume V? $\left(S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3\right)$

(A) 10π (B) 12π (C) 22.5π (D) 25π (E) 30π

EX) The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$. If the radius and the height both increase at a constant rate of $\frac{1}{2}$ centimeter per second, at what rate, in cubic centimeters per second, is the volume increasing when the height is 9 centimeters and the radius is 6 centimeters?

(A) $\frac{1}{2}\pi$ (B) 10π (C) 24π (D) 54π (E) 108π

REMINDER: Direct/Inverse Variation (from Algebra II)

Directly varies/proportional (as one variable increases, so does the other) means $y = kx$ where k is a constant

Inversely varies/proportional (as one variable increases, the other decreases) means $y = k/x$ where k is a constant

AP Examples Using Variation

The weight of a population of yeast is given by a differentiable function, W, where $W(t)$ is measured in grams and t is measured in hours. The weight of the yeast population increases with respect to t at a rate that is directly proportional to the weight. At time $t = 10$ hours, the weight of the yeast is 200 grams and is increasing at the rate of 5 grams per hour. Which of the following is a differential equation that models this situation?

(A) $W = 5(t - 10) + 200$

(B) $\frac{dW}{dt} = \frac{1}{2}t$

(C) $\frac{dW}{dt} = \frac{1}{40}W$

(D) $\frac{dW}{dt} = \frac{1}{2}W$

If the pressure P applied to a gas is increased while the gas is held at a constant temperature, then the volume V of the gas will decrease. The rate of change of the volume of gas with respect to the pressure is proportional to the reciprocal of the square of the pressure. Which of the following is a differential equation that could describe this relationship?

(A) $V = \frac{k}{P^2}$, where k is a positive constant.

(B) $\frac{dV}{dP} = \frac{k}{P^2}$, where k is a positive constant.

(C) $\frac{dV}{dP} = \frac{k}{P^2}$, where k is a negative constant.

(D) $\frac{dP}{dV} = \frac{k}{P^2}$, where k is a negative constant.

Right Triangle Types

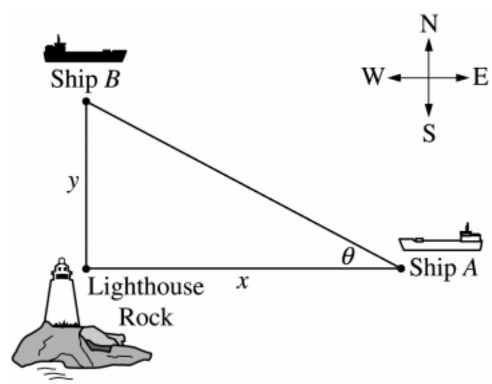
Possible Formulas to know and use:

$$(\text{leg})^2+(\text{leg})^2=(\text{hypotenuse})^2$$

$$\tan\theta=\frac{\text{opposite}}{\text{adjacent}},\ \sin\theta=\frac{\text{opposite}}{\text{hypotenuse}},\ \text{etc.}$$

$$A=\frac{1}{2}b\bullet h$$

EX)



Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure above.

(a) Find the distance, in kilometers, between Ship A and Ship B when $x = 4$ km and $y = 3$ km.

(b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.

(c) Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when $x = 4$ km and $y = 3$ km.

EX) The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?

(A) $-\frac{7}{8}$ feet per minute

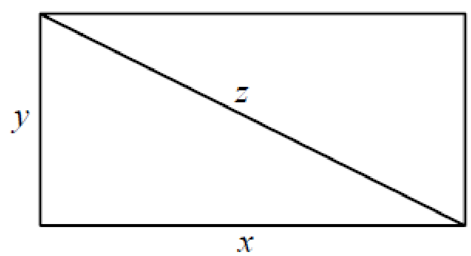
(B) $-\frac{7}{24}$ feet per minute

(C) $\frac{7}{24}$ feet per minute

(D) $\frac{7}{8}$ feet per minute

(E) $\frac{21}{25}$ feet per minute

EX)



The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$. At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5