EXERCISES FOR SECTION 2.5

In Exercises 1–16, find dy/dx by implicit differentiation.

1. $x^2 + y^2 = 16$
2. $x^2 - y^2 = 16$
3. $x^{1/2} + y^{1/2} = 9$
4. $x^3 + y^3 = 8$
5. $x^3 - xy + y^2 = 4$
6. $x^2y + y^2x = -2$
7. $x^3y^3 - y = x$
8. $\sqrt{xy} = x - 2y$
9. $x^3 - 2x^2y + 3xy^2 = 38$
10. $2 \sin x \cos y = 1$
11. $\sin x + 2 \cos 2y = 1$
12. $(\sin \pi x + \cos \pi y)^2 = 2$
13. $\sin x = x(1 + \tan y)$
14. $\cot y = x - y$
15. $y = \sin(xy)$
16. $x = \sec \dfrac{1}{y}$

In Exercises 17–24, find dy/dx by implicit differentiation and evaluate the derivative at the indicated point.

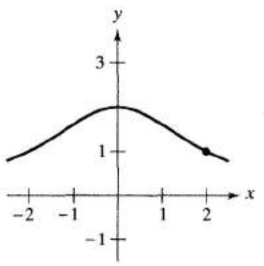
	Equation	*Point*
17.	$xy = 4$	$(-4, -1)$
18.	$x^2 - y^3 = 0$	$(1, 1)$
19.	$y^2 = \dfrac{x^2 - 9}{x^2 + 9}$	$(3, 0)$
20.	$(x + y)^3 = x^3 + y^3$	$(-1, 1)$
21.	$x^{2/3} + y^{2/3} = 5$	$(8, 1)$
22.	$x^3 + y^3 = 2xy$	$(1, 1)$
23.	$\tan(x + y) = x$	$(0, 0)$
24.	$x \cos y = 1$	$\left(2, \dfrac{\pi}{3}\right)$

In Exercises 25 and 26, use a graphing utility to graph the equation. Find an equation of the tangent line to the graph at the indicated point and sketch its graph.

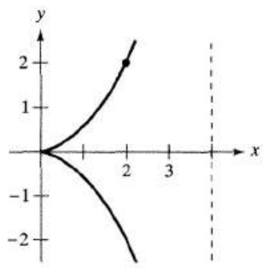
25. $\sqrt{x} + \sqrt{y} = 3, \quad (4, 1)$
26. $y^2 = \dfrac{x - 1}{x^2 + 1}, \quad \left(2, \dfrac{\sqrt{5}}{5}\right)$

In Exercises 27–30, find the slope of the tangent line to the graph at the indicated point.

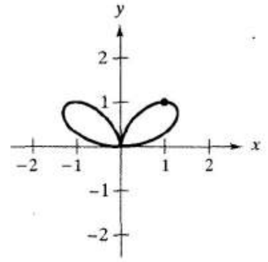
27. Witch of Agnesi:
$(x^2 + 4)y = 8$
Point: $(2, 1)$



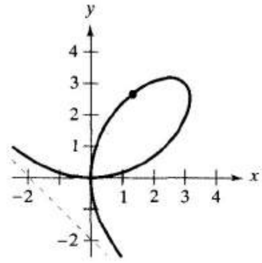
28. Cissoid:
$(4 - x)y^2 = x^3$
Point: $(2, 2)$



29. Bifolium:
$(x^2 + y^2)^2 = 4x^2y$
Point: $(1, 1)$



30. Folium of Descartes:
$x^3 + y^3 - 6xy = 0$
Point: $\left(\frac{4}{3}, \frac{8}{3}\right)$



In Exercises 31–34, (a) find two explicit functions by solving the equation for y in terms of x, (b) sketch the graph of the equation and label the parts given by the corresponding explicit functions, (c) differentiate the explicit functions, and (d) find dy/dx implicitly and show that the result is equivalent to that of part (c).

31. $x^2 + y^2 = 16$
32. $x^2 + y^2 - 4x + 6y + 9 = 0$
33. $9x^2 + 16y^2 = 144$
34. $4y^2 - x^2 = 4$

In Exercises 35–40, find d^2y/dx^2 in terms of x and y.

35. $x^2 + xy = 5$
36. $x^2y^2 - 2x = 3$
37. $x^2 - y^2 = 16$
38. $1 - xy = x - y$
39. $y^2 = x^3$
40. $y^2 = 4x$

In Exercises 41 and 42, find equations for the tangent line and normal line to the circle at the indicated points. (The *normal line* at a point is perpendicular to the tangent line at the point.) Use a graphing utility to graph the equation, tangent line, and normal line.

41. $x^2 + y^2 = 25$
$(4, 3), (-3, 4)$
42. $x^2 + y^2 = 9$
$(0, 3), (2, \sqrt{5})$

43. Show that the normal line at any point on the circle $x^2 + y^2 = r^2$ passes through the origin.
44. Two circles of radius 4 are tangent to the graph of $y^2 = 4x$ at the point $(1, 2)$. Find the equations of these two circles.

In Exercises 45 and 46, find the points at which the graph of the equation has a vertical or horizontal tangent line.

45. $25x^2 + 16y^2 + 200x - 160y + 400 = 0$
46. $4x^2 + y^2 - 8x + 4y + 4 = 0$

Orthogonal Trajectories **In Exercises 47–50, use a graphing utility to sketch the intersecting graphs of the equations and show that they are orthogonal. [Two graphs are *orthogonal* if at their point(s) of intersection their tangent lines are perpendicular to each other.]**

47. $2x^2 + y^2 = 6$
$y^2 = 4x$

48. $y^2 = x^3$
$2x^2 + 3y^2 = 5$

49. $x + y = 0$
$x = \sin y$

50. $x^3 = 3(y - 1)$
$x(3y - 29) = 3$

Orthogonal Trajectories **In Exercises 51 and 52, verify that the two families of curves are orthogonal where C and K are real numbers. Use a graphing utility to graph the two families for two values of C and two values of K.**

51. $xy = C$
$x^2 - y^2 = K$

52. $x^2 + y^2 = C^2$
$y = Kx$

In Exercises 53–56, differentiate (a) with respect to x (y is a function of x) and (b) with respect to t (x and y are functions of t).

53. $2y^2 - 3x^4 = 0$

54. $x^2 - 3xy^2 + y^3 = 10$

55. $\cos \pi y - 3 \sin \pi x = 1$

56. $4 \sin x \cos y = 1$

57. Consider the equation $x^4 = 4(4x^2 - y^2)$.

(a) Use a graphing utility to graph the equation.

(b) Find and graph the four tangent lines to the curve for $y = 3$.

(c) Find the exact coordinates of the point of intersection of the two tangent lines in the first quadrant.

58. Use Example 6 as a model to find dy/dx implicitly for the equation $\tan y = x$ and find the largest interval of the form $-a < y < a$ such that y is a differentiable function of x. Then express dy/dx as a function of x.

59. Prove (Theorem 2.3) that

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

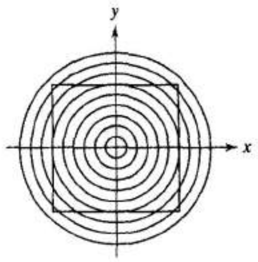
for the case in which n is a rational number. (*Hint:* Write $y = x^{p/q}$ in the form $y^q = x^p$ and differentiate implicitly. Assume that p and q are integers, where $q > 0$.)

60. Let L be any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$. Show that the sum of the x- and y-intercepts of L is c.

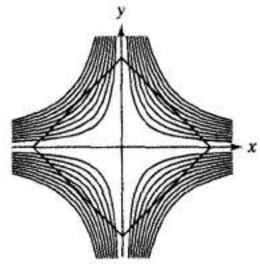
SECTION PROJECT

Optical Illusions **In each graph below, an optical illusion is created by having lines intersect a family of curves. In each case, the lines appear to be curved. Find the value of dy/dx for the indicated values of x and y.**

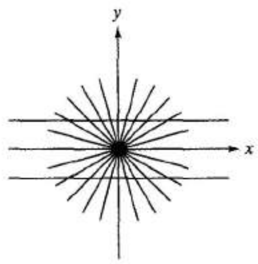
(a) Circles: $x^2 + y^2 = C^2$
$x = 3, y = 4, C = 5$



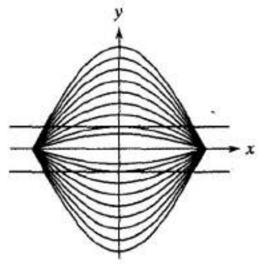
(b) Hyperbolas: $xy = C$
$x = 1, y = 4, C = 4$



(c) Lines: $ax = by$
$x = \sqrt{3}, y = 3,$
$a = \sqrt{3}, b = 1$



(d) Cosine curves: $y = C \cos x$
$x = \frac{\pi}{3}, y = \frac{1}{3}, C = \frac{2}{3}$



FOR FURTHER INFORMATION For more information on the mathematics of optical illusions, see the article "Descriptive Models for Perception of Optical Illusions" by David A. Smith in the 1985 (Volume 2) issue of the *UMAP Journal*.