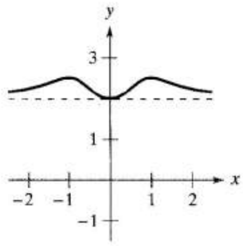
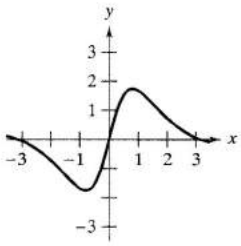
EXERCISES FOR SECTION 3.5

In Exercises 1–6, match the function with one of the graphs [(a), (b), (c), (d), (e), or (f)] using horizontal asymptotes as an aid.

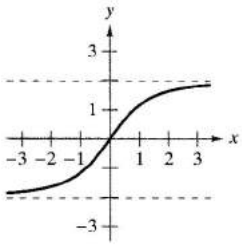
(a)



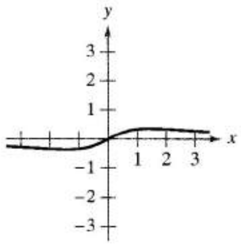
(b)



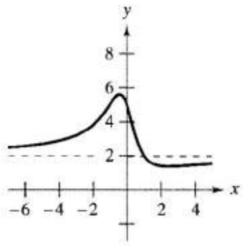
(c)



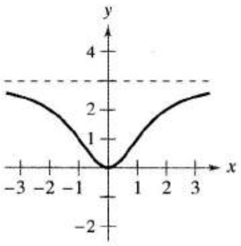
(d)



(e)



(f)



1. $f(x) = \dfrac{3x^2}{x^2 + 2}$ **2.** $f(x) = \dfrac{2x}{\sqrt{x^2 + 2}}$

3. $f(x) = \dfrac{x}{x^2 + 2}$ **4.** $f(x) = 2 + \dfrac{x^2}{x^4 + 1}$

5. $f(x) = \dfrac{4 \sin x}{x^2 + 1}$ **6.** $f(x) = \dfrac{2x^2 - 3x + 5}{x^2 + 1}$

Numerical and Graphical Analysis **In Exercises 7–10, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

7. $f(x) = \dfrac{4x + 3}{2x - 1}$ **8.** $f(x) = \dfrac{2x^2}{x + 1}$

9. $f(x) = \dfrac{-6x}{\sqrt{4x^2 + 5}}$ **10.** $f(x) = 5 - \dfrac{1}{x^2 + 1}$

In Exercises 11–24, find the limit.

11. $\lim_{x \to \infty} \dfrac{2x - 1}{3x + 2}$ **12.** $\lim_{x \to \infty} \dfrac{5x^3 + 1}{10x^3 - 3x^2 + 7}$

13. $\lim_{x \to \infty} \dfrac{x}{x^2 - 1}$ **14.** $\lim_{x \to \infty} \dfrac{2x^{10} - 1}{10x^{11} - 3}$

15. $\lim_{x \to -\infty} \dfrac{5x^2}{x + 3}$ **16.** $\lim_{x \to \infty} \left(2x - \dfrac{1}{x^2}\right)$

17. $\lim_{x \to -\infty} \dfrac{x}{\sqrt{x^2 - x}}$ **18.** $\lim_{x \to \infty} \dfrac{x}{\sqrt{x^2 + 1}}$

19. $\lim_{x \to \infty} \dfrac{2x + 1}{\sqrt{x^2 - x}}$ **20.** $\lim_{x \to -\infty} \dfrac{-3x + 1}{\sqrt{x^2 + x}}$

21. $\lim_{x \to \infty} \dfrac{\sin 2x}{x}$ **22.** $\lim_{x \to \infty} \dfrac{x - \cos x}{x}$

23. $\lim_{x \to \infty} \dfrac{1}{2x + \sin x}$ **24.** $\lim_{x \to \infty} \sin \dfrac{1}{x}$

In Exercises 25 and 26, find the limit. (*Hint:* Let $x = 1/t$ and find the limit as $t \to 0^+$.)

25. $\lim_{x \to \infty} x \sin \dfrac{1}{x}$ **26.** $\lim_{x \to \infty} x \tan \dfrac{1}{x}$

In Exercises 27–30, find the limit (*Hint:* Treat the expression as a fraction whose denominator is 1, and rationalize the numerator.) Use a graphing utility to verify your result.

27. $\lim_{x \to -\infty} \left(x + \sqrt{x^2 + 3}\right)$

28. $\lim_{x \to \infty} \left(2x - \sqrt{4x^2 + 1}\right)$

29. $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x}\right)$

30. $\lim_{x \to -\infty} \left(3x + \sqrt{9x^2 - x}\right)$

Numerical, Graphical, and Analytic Analysis **In Exercises 31–34, use a graphing utility to complete the table and estimate the limit as x approaches infinity. Then use a graphing utility to graph the function and estimate the limit graphically. Finally, find the limit analytically and compare your results with the estimates.**

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

31. $f(x) = x - \sqrt{x(x - 1)}$ **32.** $f(x) = x^2 - x\sqrt{x(x - 1)}$

33. $f(x) = x \sin \dfrac{1}{2x}$ **34.** $f(x) = \dfrac{x + 1}{x\sqrt{x}}$

In Exercises 35–52, sketch the graph of the equation. Look for extrema, intercepts, symmetry, and asymptotes as necessary. Use a graphing utility to verify your result.

35. $y = \dfrac{2 + x}{1 - x}$

36. $y = \dfrac{x - 3}{x - 2}$

37. $y = \dfrac{x}{x^2 - 4}$

38. $y = \dfrac{2x}{9 - x^2}$

39. $y = \dfrac{x^2}{x^2 + 9}$

40. $y = \dfrac{x^2}{x^2 - 9}$

41. $y = \dfrac{2x^2}{x^2 - 4}$

42. $y = \dfrac{2x^2}{x^2 + 4}$

43. $xy^2 = 4$

44. $x^2y = 4$

45. $y = \dfrac{2x}{1 - x}$

46. $y = \dfrac{2x}{1 - x^2}$

47. $y = 2 - \dfrac{3}{x^2}$

48. $y = 1 + \dfrac{1}{x}$

49. $y = 3 + \dfrac{2}{x}$

50. $y = 4\left(1 - \dfrac{1}{x^2}\right)$

51. $y = \dfrac{x^3}{\sqrt{x^2 - 4}}$

52. $y = \dfrac{x}{\sqrt{x^2 - 4}}$

In Exercises 53–60, use a symbolic differentiation utility to analyze the graph of the function. Label any extrema and/or asymptotes that exist.

53. $f(x) = 5 - \dfrac{1}{x^2}$

54. $f(x) = \dfrac{x^2}{x^2 - 1}$

55. $f(x) = \dfrac{x}{x^2 - 4}$

56. $f(x) = \dfrac{1}{x^2 - x - 2}$

57. $f(x) = \dfrac{x - 2}{x^2 - 4x + 3}$

58. $f(x) = \dfrac{x + 1}{x^2 + x + 1}$

59. $f(x) = \dfrac{3x}{\sqrt{4x^2 + 1}}$

60. $f(x) = \dfrac{2 \sin 2x}{x}$

61. Use a graphing utility to graph each function and verify that each has two horizontal asymptotes.

(a) $f(x) = \dfrac{|x|}{x + 1}$ (b) $f(x) = \dfrac{2x}{\sqrt{x^2 + 1}}$

62. Given the function $f(x) = 5x^3 - 3x^2 + 10$, find $\lim_{x\to\infty} h(x)$, if possible.

(a) $h(x) = \dfrac{f(x)}{x^2}$ (b) $h(x) = \dfrac{f(x)}{x^3}$ (c) $h(x) = \dfrac{f(x)}{x^4}$

In Exercises 63 and 64, (a) use a graphing utility to graph f and g in the same viewing rectangle, (b) verify algebraically that f and g represent the same function, and (c) zoom out sufficiently far so that the graph appears as a line. What equation does this line appear to have? (Note that the points at which the function is not continuous are not readily seen when you zoom out.)

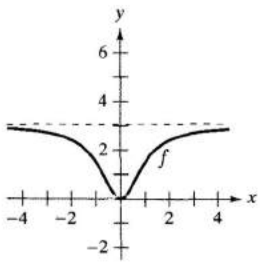
63. $f(x) = \dfrac{x^3 - 3x^2 + 2}{x(x - 3)}$

$g(x) = x + \dfrac{2}{x(x - 3)}$

64. $f(x) = -\dfrac{x^3 - 2x^2 + 2}{2x^2}$

$g(x) = -\dfrac{1}{2}x + 1 - \dfrac{1}{x^2}$

65. ***Think About It*** The graph of a function f is shown below.



(a) Sketch f'.

(b) Use the graphs to estimate $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} f'(x)$.

(c) Explain the answers you gave in part (b).

66. ***Engine Efficiency*** The efficiency of an internal combustion engine is

$$\text{Efficiency } (\%) = 100\left[1 - \frac{1}{(v_1/v_2)^c}\right]$$

where v_1/v_2 is the ratio of the uncompressed gas to the compressed gas and c is a positive constant dependent on the engine design. Find the limit of the efficiency as the compression ratio approaches infinity.

67. ***Average Cost*** A business has a cost of $C = 0.5x + 500$ for producing x units. The average cost per unit is

$$\overline{C} = \frac{C}{x}.$$

Find the limit of $\overline{C}$ as x approaches infinity.

68. A line with slope m passes through the point $(0, 4)$.

(a) Write the distance d between the line and the point $(3, 1)$ as a function of m.

(b) Use a graphing utility to graph the equation in part (a).

(c) Find $\lim_{m\to\infty} d(m)$ and $\lim_{m\to-\infty} d(m)$. Interpret the results geometrically.

69. *Modeling Data* A heat probe is attached to the heat exchanger of a heating system. The temperature T (degrees Celsius) is recorded t seconds after the furnace is started. The results for the first 2 minutes are recorded in the table.

t	0	15	30	45	60	75	90	105	120
T	25.2°	36.9°	45.5°	51.4°	56.0°	59.6°	62.0°	64.0°	65.2°

(a) Use the regression capabilities of a graphing utility to find a model of the form $T_1 = at^2 + bt + c$ for the data.

(b) Use a graphing utility to graph T_1.

(c) A rational model for the data is

$$T_2 = \frac{1451 + 86t}{58 + t}.$$

Use a graphing utility to graph the model.

(d) Find $T_1(0)$ and $T_2(0)$.

(e) Find $\lim_{t \to \infty} T_2$.

(f) Interpret the result in part (e) in the context of the problem. Is it possible to do this type of analysis using T_1? Explain.

70. *Modeling Data* The data in the table give the number N (in thousands) of high school graduates at the end of each decade for the years 1900 through 1990. *(Source: U.S. Department of Education)*

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990
N	62	111	231	592	1140	1063	1627	2589	2748	2505

A model for these data is

$$N = \frac{68{,}436.82 + 4731.82t}{1000 - 23.37t + 0.16t^2}, \quad 0 \le t \le 90$$

where t is the time in years, with $t = 0$ corresponding to 1990.

(a) Use a graphing utility to plot the data and graph the model.

(b) Use the model to estimate the number of high school graduates in 1975.

(c) Approximate the year when the number of graduates was greatest.

(d) Use a symbolic differentiation utility to determine the time when the rate of increase in the number of graduates was greatest.

(e) Why should this model *not* be used to predict the number of graduates in future years?

True or False? **In Exercises 71 and 72, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

71. If $f'(x) > 0$ for all real numbers x, then f increases without bound.

72. If $f''(x) < 0$ for all real numbers x, then f decreases without bound.

73. *Think About It* Sketch a graph of a differentiable function f that satisfies the following conditions and has $x = 2$ as its only critical number.

$f'(x) < 0$ for $x < 2$

$f'(x) > 0$ for $x > 2$

$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} f(x) = 6$

74. *Think About It* Is it possible to sketch a graph of a function that satisfies the conditions of Exercise 73 and has *no* points of inflection? Explain.

75. Prove that if $p(x) = a_n x^n + \cdots + a_1 x + a_0$ and $q(x) = b_m x^m + \cdots + b_1 x + b_0$ $(a_n \ne 0, b_m \ne 0)$, then

$$\lim_{x \to \infty} \frac{p(x)}{q(x)} = \begin{cases} 0, & n < m \\ \dfrac{a_n}{b_m}, & n = m \\ \pm\infty, & n > m. \end{cases}$$

SECTION PROJECT

Graph the functions

$$f(x) = \frac{2x^2 - x}{x - 1} \quad \text{and} \quad g(x) = 2x + 1$$

in the same viewing rectangle, and zoom out a few times.

(a) Describe the behavior of the graphs of f and g as $x \to \infty$ and $x \to -\infty$.

(b) Use long division of polynomials to show that

$$f(x) = 2x + 1 + \frac{1}{x - 1}.$$

What does this mean about the shapes of the graphs of f and g as $x \to \infty$ and $x \to -\infty$?

(c) Show that f can also be written as follows.

$$f(x) = \frac{2x - 1}{1 - 1/x}.$$

Does this mean that the graph of f should resemble that of $h(x) = 2x - 1$ as $x \to \infty$ and $x \to -\infty$? Explain.

(d) Apply the techniques above to write a short paragraph about the shape of the graph of

$$y = \frac{1 + 2x - 2x^2}{2x}$$

as $x \to \infty$ and $x \to -\infty$.