EXERCISES FOR SECTION 5.1

1. Complete the table below. Use a graphing utility and Simpson's Rule with $n = 10$ to approximate the integral

$$\int_1^x \frac{1}{t}\,dt.$$

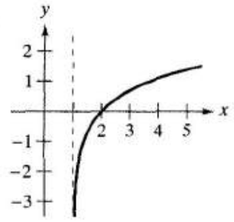
x	0.5	1.5	2	2.5	3	3.5	4
$\int_1^x (1/t)\,dt$							

2. (a) Plot the points generated in Exercise 1 and connect them with a smooth curve. Compare the result with the graph of $y = \ln x$.

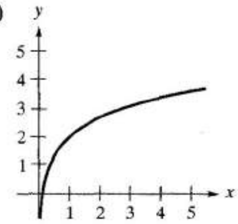
(b) Use a graphing utility to graph $y = \int_1^x (1/t)\,dt$ for $0.2 \le x \le 4$. Compare the result with the graph of $y = \ln x$.

In Exercises 3–6, use the graph of $y = \ln x$ to match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a)



(b)



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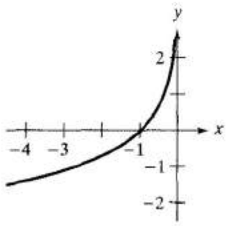
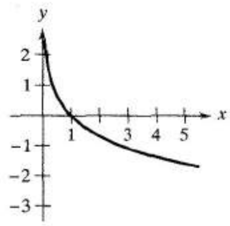


(d)



3. $f(x) = \ln x + 2$

4. $f(x) = -\ln x$

5. $f(x) = \ln(x - 1)$

6. $f(x) = -\ln(-x)$

In Exercises 7–12, sketch the graph of the function and state its domain.

7. $f(x) = 3 \ln x$

8. $f(x) = -2 \ln x$

9. $f(x) = \ln 2x$

10. $f(x) = \ln|x|$

11. $f(x) = \ln(x - 1)$

12. $g(x) = 2 + \ln x$

In Exercises 13 and 14, use the properties of logarithms to approximate the indicated logarithm, given that $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$.

13. (a) $\ln 6$ (b) $\ln \frac{2}{3}$ (c) $\ln 81$ (d) $\ln \sqrt{3}$

14. (a) $\ln 0.25$ (b) $\ln 24$ (c) $\ln \sqrt[3]{12}$ (d) $\ln \frac{1}{72}$

In Exercises 15–24, use the properties of logarithms to write the expression as a sum, difference, and/or multiple of logarithms.

15. $\ln \frac{2}{3}$

16. $\ln \frac{1}{5}$

17. $\ln \dfrac{xy}{z}$

18. $\ln(xyz)$

19. $\ln \sqrt{2^3}$

20. $\ln \sqrt{a - 1}$

21. $\ln\left(\dfrac{x^2 - 1}{x^3}\right)^3$

22. $\ln 3e^2$

23. $\ln z(z - 1)^2$

24. $\ln \dfrac{1}{e}$

In Exercises 25–30, write the expression as a logarithm of a single quantity.

25. $\ln(x - 2) - \ln(x + 2)$

26. $3 \ln x + 2 \ln y - 4 \ln z$

27. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

28. $2[\ln x - \ln(x + 1) - \ln(x - 1)]$

29. $2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$

30. $\frac{3}{2}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

In Exercises 31 and 32, show that $f = g$ by using a graphing utility to graph f and g in the same viewing rectangle.

31. $f(x) = \ln \dfrac{x^2}{4},\ x > 0,\quad g(x) = 2 \ln x - \ln 4$

32. $f(x) = \ln \sqrt{x(x^2 + 1)},\quad g(x) = \frac{1}{2}[\ln x + \ln(x^2 + 1)]$

In Exercises 33–36, find the limit.

33. $\lim\limits_{x \to 3^+} \ln(x - 3)$

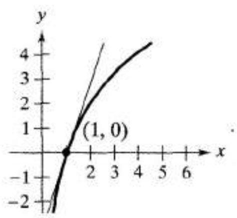
34. $\lim\limits_{x \to 6^-} \ln(6 - x)$

35. $\lim\limits_{x \to 2^-} \ln[x^2(3 - x)]$

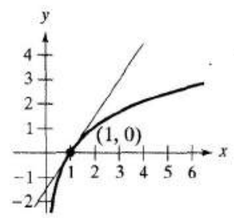
36. $\lim\limits_{x \to 5^+} \ln \dfrac{x}{\sqrt{x - 4}}$

In Exercises 37–40, find the slope of the tangent line to the logarithmic function at the point (1, 0).

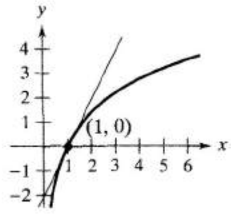
37. $y = \ln x^3$



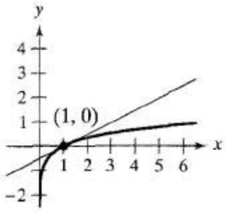
38. $y = \ln x^{3/2}$



39. $y = \ln x^2$



40. $y = \ln x^{1/2}$



In Exercises 41–66, find the derivative of the function.

41. $g(x) = \ln x^2$

42. $h(x) = \ln(x^2 + 3)$

43. $y = (\ln x)^4$

44. $y = x \ln x$

45. $y = \ln\left(x\sqrt{x^2 - 1}\right)$

46. $y = \ln\sqrt{x^2 - 4}$

47. $f(x) = \ln\left(\dfrac{x}{x^2 + 1}\right)$

48. $f(x) = \ln\left(\dfrac{x}{x + 1}\right)$

49. $g(t) = \dfrac{\ln t}{t^2}$

50. $h(t) = \dfrac{\ln t}{t}$

51. $y = \ln(\ln x^2)$

52. $y = \ln(\ln x)$

53. $y = \ln\sqrt{\dfrac{x + 1}{x - 1}}$

54. $y = \ln\sqrt{\dfrac{x - 1}{x + 1}}$

55. $f(x) = \ln\left(\dfrac{\sqrt{4 + x^2}}{x}\right)$

56. $f(x) = \ln\left(x + \sqrt{4 + x^2}\right)$

57. $y = \dfrac{-\sqrt{x^2 + 1}}{x} + \ln\left(x + \sqrt{x^2 + 1}\right)$

58. $y = \dfrac{-\sqrt{x^2 + 4}}{2x^2} - \dfrac{1}{4}\ln\left(\dfrac{2 + \sqrt{x^2 + 4}}{x}\right)$

59. $y = \ln|\sin x|$

60. $y = \ln|\sec x|$

61. $y = \ln\left|\dfrac{\cos x}{\cos x - 1}\right|$

62. $y = \ln|\sec x + \tan x|$

63. $y = \ln\left|\dfrac{-1 + \sin x}{2 + \sin x}\right|$

64. $y = \ln\sqrt{1 + \sin^2 x}$

65. $f(x) = \sin 2x \ln x^2$

66. $g(x) = \displaystyle\int_1^{\ln x} (t^2 + 3)\,dt$

In Exercises 67 and 68, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the derivative feature of a graphing utility to confirm your results.

Function	*Point*
67. $y = 3x^2 - \ln x$	$(1, 3)$
68. $y = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right)$	$(0, 4)$

In Exercises 69 and 70, use implicit differentiation to find dy/dx.

69. $x^2 - 3\ln y + y^2 = 10$

70. $\ln xy + 5x = 30$

In Exercises 71 and 72, show that the function is a solution of the differential equation.

Function	*Differential Equation*
71. $y = 2\ln x + 3$	$xy'' + y' = 0$
72. $y = x\ln x - 4x$	$x + y - xy' = 0$

In Exercises 73–78, find any relative extrema and inflection points of the function. Use a graphing utility to confirm your results.

73. $y = \dfrac{x^2}{2} - \ln x$

74. $y = x - \ln x$

75. $y = x \ln x$

76. $y = \dfrac{\ln x}{x}$

77. $y = \dfrac{x}{\ln x}$

78. $y = x^2 \ln x$

Linear and Quadratic Approximations **In Exercises 79 and 80, use a graphing utility to graph the function. Then graph**

$$P_1(x) = f(1) + f'(1)(x - 1)$$

and

$$P_2(x) = f(1) + f'(1)(x - 1) + \tfrac{1}{2}f''(1)(x - 1)^2$$

in the same viewing rectangle. Compare the values of f, P_1, and P_2 and their first derivatives at $x = 1$.

79. $f(x) = \ln x$

80. $f(x) = x \ln x$

In Exercises 81 and 82, use Newton's Method to approximate, to three decimal places, the x-coordinate of the point of intersection of the graphs of the two equations. Use a graphing utility to verify your result.

81. $y = \ln x$
$y = -x$

82. $y = \ln x$
$y = 3 - x$

In Exercises 83–88, find dy/dx using logarithmic differentiation.

83. $y = x\sqrt{x^2 - 1}$

84. $y = \sqrt{(x - 1)(x - 2)(x - 3)}$

85. $y = \dfrac{x^2\sqrt{3x - 2}}{(x - 1)^2}$

86. $y = \sqrt[3]{\dfrac{x^2 + 1}{x^2 - 1}}$

87. $y = \dfrac{x(x - 1)^{3/2}}{\sqrt{x + 1}}$

88. $y = \dfrac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

EXERCISES FOR SECTION 5.4

In Exercises 1–4, write the exponential equation as a logarithmic equation or vice versa.

1. $e^0 = 1$
2. $e^{-2} = 0.1353\ldots$
3. $\ln 2 = 0.6931\ldots$
4. $\ln 0.5 = -0.6931\ldots$

In Exercises 5–8, solve for x.

5. $e^{\ln x} = 4$
6. $e^{\ln 2x} = 12$
7. $\ln x = 2$
8. $\ln x^2 = 10$

In Exercises 9–12, sketch the graph of the function.

9. $y = e^{-x}$
10. $y = \frac{1}{2}e^x$
11. $y = e^{-x^2}$
12. $y = e^{-x/2}$

13. Use a graphing utility to graph $f(x) = e^x$ and the given function in the same viewing rectangle. How are the two graphs related?
(a) $g(x) = e^{x-2}$ (b) $h(x) = -\frac{1}{2}e^x$ (c) $q(x) = e^{-x} + 3$

14. Use a graphing utility to graph the function. Use the graph to determine any asymptotes of the function.

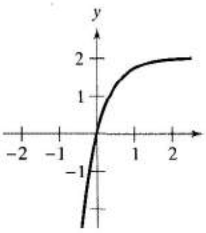
(a) $f(x) = \dfrac{8}{1 + e^{-0.5x}}$ (b) $g(x) = \dfrac{8}{1 + e^{-0.5/x}}$

In Exercises 15–18, illustrate that the functions are inverses of each other by graphing both functions on the same set of coordinate axes.

15. $f(x) = e^{2x}$
$g(x) = \ln \sqrt{x}$
16. $f(x) = e^{x/3}$
$g(x) = \ln x^3$
17. $f(x) = e^x - 1$
$g(x) = \ln(x + 1)$
18. $f(x) = e^{x-1}$
$g(x) = 1 + \ln x$

In Exercises 19–22, match the equation with the correct graph. Assume that a and C are arbitrary real numbers such that $a > 0$. [The graphs are labeled (a), (b), (c), and (d).]

(a)



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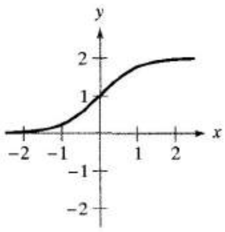

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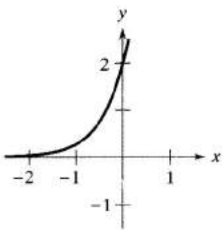

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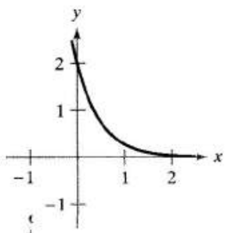


19. $y = Ce^{ax}$
20. $y = Ce^{-ax}$
21. $y = C(1 - e^{-ax})$
22. $y = \dfrac{C}{1 + e^{-ax}}$

23. ***Graphical Analysis*** Use a graphing utility to graph

$$f(x) = \left(1 + \frac{0.5}{x}\right)^x \quad \text{and} \quad g(x) = e^{0.5}$$

in the same viewing rectangle. What is the relationship between f and g as $x \to \infty$?

24. ***Conjecture*** Use the result of Exercise 23 to make a conjecture about the value of

$$\left(1 + \frac{r}{x}\right)^x$$

as $x \to \infty$.

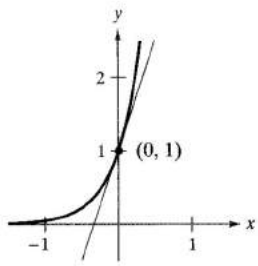
In Exercises 25 and 26, compare the given number with the number e. Is the number less than or greater than e?

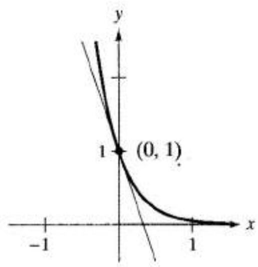
25. $\left(1 + \dfrac{1}{1{,}000{,}000}\right)^{1{,}000{,}000}$ (See Exercise 24.)

26. $1 + 1 + \dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{24} + \dfrac{1}{120} + \dfrac{1}{720} + \dfrac{1}{5040}$

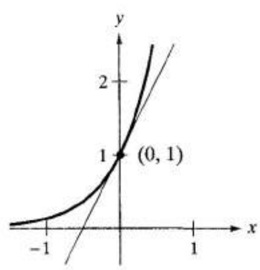
In Exercises 27 and 28, find the slope of the tangent line to the function at the point (0, 1).

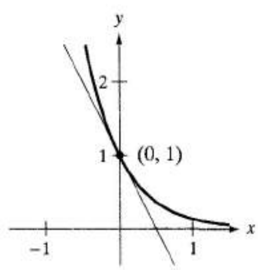
27. (a) $y = e^{3x}$ (b) $y = e^{-3x}$





28. (a) $y = e^{2x}$ (b) $y = e^{-2x}$





In Exercises 29–48, find the derivative of the function.

29. $f(x) = e^{2x}$
30. $f(x) = e^{1-x}$
31. $y = e^{-2x+x^2}$
32. $y = e^{-x^2}$
33. $y = e^{\sqrt{x}}$
34. $y = x^2e^{-x}$
35. $g(t) = (e^{-t} + e^t)^3$
36. $g(t) = e^{-1/t^2}$
37. $y = \ln(e^{x^2})$
38. $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$
39. $y = \ln(1 + e^{2x})$
40. $y = \ln\frac{e^x + e^{-x}}{2}$
41. $y = \frac{2}{e^x + e^{-x}}$
42. $y = \frac{e^x - e^{-x}}{2}$
43. $y = x^2e^x - 2xe^x + 2e^x$
44. $y = xe^x - e^x$
45. $f(x) = e^{-x}\ln x$
46. $f(x) = e^3 \ln x$
47. $y = e^x(\sin x + \cos x)$
48. $y = \ln e^x$

In Exercises 49 and 50, use implicit differentiation to find dy/dx.

49. $xe^y - 10x + 3y = 0$
50. $e^{xy} + x^2 - y^2 = 10$

In Exercises 51 and 52, find the second derivative of the function.

51. $f(x) = (3 + 2x)e^{-3x}$
52. $g(x) = \sqrt{x} + e^x \ln x$

In Exercises 53 and 54, show that the function $y = f(x)$ is a solution of the differential equation.

53. $y = e^x(\cos\sqrt{2}x + \sin\sqrt{2}x)$

$y'' - 2y' + 3y = 0$

54. $y = e^x(3\cos 2x - 4\sin 2x)$

$y'' - 2y' + 5y = 0$

In Exercises 55–60, find the extrema and the points of inflection (if any exist). Use a graphing utility to graph the function and confirm your results.

55. $f(x) = \frac{1}{\sqrt{2\pi}}e^{-(x^2/2)}$
56. $f(x) = \frac{e^x - e^{-x}}{2}$
57. $f(x) = \frac{e^x + e^{-x}}{2}$
58. $f(x) = xe^{-x}$
59. $f(x) = x^2e^{-x}$
60. $f(x) = -2 + e^{3x}(4 - 2x)$

61. ***Area*** Find the area of the largest rectangle that can be inscribed under the curve $y = e^{-x^2}$ in the first and second quadrants.

62. ***Area*** Perform the following steps to find the maximum area of the rectangle shown in the figure.

(a) Solve for c in the equation $f(c) = f(c + x)$.

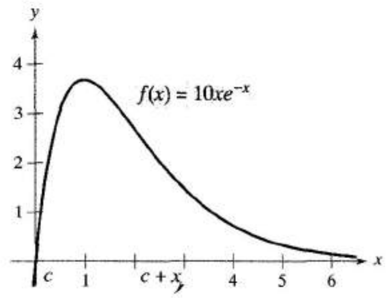
(b) Use the result in part (a) to write the area A as a function of x. [*Hint:* $A = xf(c)$]

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions of the rectangle of maximum area. Determine the required area.

(d) Use a graphing utility to graph the expression for c found in part (a). Use the graph to approximate

$$\lim_{x\to 0^+} c \quad \text{and} \quad \lim_{x\to\infty} c.$$

Use this result to describe the changes in dimensions and position of the rectangle for $0 < x < \infty$.



63. Verify that the function

$$y = \frac{L}{1 + ae^{-x/b}}, \qquad a > 0, b > 0, L > 0$$

increases at a maximum rate when $y = L/2$.

64. Find the point on the graph of $y = e^{-x}$ where the normal line to the curve passes through the origin. (Use Newton's Method or the root-finding capabilities of a graphing utility.)

65. Find, to three decimal places, the value of x such that

$$e^{-x} = x.$$

(Use Newton's Method or the root-finding capabilities of a graphing utility.)

66. ***Depreciation*** The value V of an item t years after it is purchased is

$$V = 15{,}000e^{-0.6286t}, \qquad 0 \le t \le 10.$$

(a) Use a graphing utility to graph the function.

(b) Find the rate of change of V with respect to t when $t = 1$ and $t = 5$.

(c) Use a graphing utility to graph the tangent line to the function when $t = 1$ and $t = 5$.

67. ***Writing*** Consider the function

$$f(x) = \frac{2}{1 + e^{1/x}}.$$

(a) Use a graphing utility to graph f.

(b) Write a short paragraph explaining why the graph has a horizontal asymptote at $y = 1$ and why the function has a nonremovable discontinuity at $x = 0$.

68. ***Harmonic Motion*** The displacement from equilibrium of a mass oscillating on the end of a spring suspended from a ceiling is

$y = 1.56e^{-0.22t} \cos 4.9t$

where y is the displacement in feet and t is the time in seconds. Use a graphing utility to graph the displacement function on the interval $[0, 10]$. Find a value of t past which the displacement is less than 3 inches from equilibrium.

69. ***Modeling Data*** A meteorologist measures the atmospheric pressure P (in kilograms per square meter) at altitude h (in kilometers). The data are shown below.

h	0	5	10	15	20
P	10,332	5583	2376	1240	517

(a) Use a graphing utility to plot the points $(h, \ln P)$. Use the regression capabilities of the graphing utility to find a linear model for the revised data points.

(b) The line in part (a) has the form $\ln P = ah + b$. Write the equation in exponential form.

(c) Use a graphing utility to plot the original data and graph the exponential model in part (b).

(d) Find the rate of change of the pressure when $h = 5$ and $h = 18$.

70. ***Modeling Data*** A 1990 Chevrolet Beretta with a six-cylinder engine, automatic transmission, and air conditioning had a retail price of \$11,500. A local dealership had the following guide for the approximate value of the car for the years 1990 through 1995. *(Source: National Automobile Dealer's Association)*

Year	1990	1991	1992
Value	\$11,500	\$9315	\$9200

Year	1993	1994	1995
Value	\$7935	\$7130	\$6095

In each of the following, let V represent the value of the automobile in the year t, with $t = 0$ corresponding to 1990.

(a) Use the regression capabilities of a graphing utility to find linear and quadratic models for the data. Use the graphing utility to plot the data and graph the models.

(b) What does the slope represent in the linear model in part (a)?

(c) Use the regression capabilities of a graphing utility to find a linear model for the points $(t, \ln V)$. Write the resulting equation of the form $\ln V = at + b$ in exponential form.

(d) Determine the horizontal asymptote of the exponential model in part (c). Interpret its meaning in the context of the problem.

(e) Find the rate of decrease in the value of the car when $t = 1$ and $t = 5$.

Linear and Quadratic Approximations **In Exercises 71 and 72, use a graphing utility to graph the function. Then graph**

$$P_1(x) = f(0) + f'(0)(x - 0)$$

and

$$P_2(x) = f(0) + f'(0)(x - 0) + \tfrac{1}{2}f''(0)(x - 0)^2$$

in the same viewing rectangle. Compare the values of f, P_1, and P_2 and their first derivatives at $x = 0$.

71. $f(x) = e^{x/2}$ **72.** $f(x) = e^{-x^2/2}$

73. ***Finding a Pattern*** Use a graphing utility to compare the graph of the function $y = e^x$ with the graphs of each of the following functions.

(a) $y_1 = 1 + \frac{x}{1!}$

(b) $y_2 = 1 + \frac{x}{1!} + \frac{x^2}{2!}$

(c) $y_3 = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$

74. Identify the pattern of successive polynomials in Exercise 73. Extend the pattern one more term and compare the graph of the resulting polynomial function with the graph of $y = e^x$. What do you think this pattern implies?

In Exercises 75–92, evaluate the integral.

75. $\int e^{5x}(5)\, dx$ **76.** $\int e^{-x^4}(-4x^3)\, dx$

77. $\int_0^1 e^{-2x}\, dx$ **78.** $\int_1^2 e^{1-x}\, dx$

79. $\int \frac{e^{-x}}{1 + e^{-x}}\, dx$ **80.** $\int \frac{e^{2x}}{1 + e^{2x}}\, dx$

81. $\int_1^3 \frac{e^{3/x}}{x^2}\, dx$ **82.** $\int_0^{\sqrt{2}} xe^{-(x^2/2)}\, dx$

83. $\int e^x\sqrt{1 - e^x}\, dx$ **84.** $\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

85. $\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$ **86.** $\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\, dx$

87. $\int \frac{5 - e^x}{e^{2x}}\, dx$ **88.** $\int \frac{e^{2x} + 2e^x + 1}{e^x}\, dx$

89. $\int e^{\sin \pi x} \cos \pi x\, dx$ **90.** $\int e^{\tan 2x} \sec^2 2x\, dx$

91. $\int e^{-x} \tan(e^{-x})\, dx$ **92.** $\int \ln(e^{2x-1})\, dx$

In Exercises 93 and 94, solve the differential equation.

93. $\frac{dy}{dx} = xe^{ax^2}$ **94.** $\frac{dy}{dx} = (e^x - e^{-x})^2$

In Exercises 95 and 96, find the particular solution of the differential equation that satisfies the initial conditions.

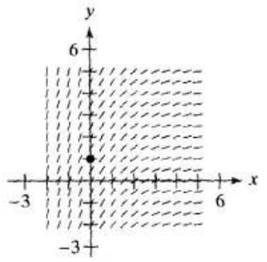
95. $f''(x) = \frac{1}{2}(e^x + e^{-x})$, $f(0) = 1, f'(0) = 0$

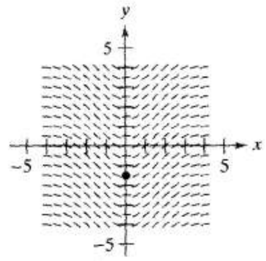
96. $f''(x) = \sin x + e^{2x}$, $f(0) = \frac{1}{4}, f'(0) = \frac{1}{2}$

Direction Fields **In Exercises 97 and 98, a differential equation, a point, and a direction field are given. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

97. $\frac{dy}{dx} = 2e^{-x/2}$, $(0, 1)$

98. $\frac{dy}{dx} = xe^{-0.2x^2}$, $\left(0, -\frac{3}{2}\right)$





Area **In Exercises 99–102, find the area of the region bounded by the graphs of the equations. Use a graphing utility to graph the region and verify your result.**

99. $y = e^x, y = 0, x = 0, x = 5$

100. $y = e^{-x}, y = 0, x = a, x = b$

101. $y = xe^{-(x^2/2)}, y = 0, x = 0, x = \sqrt{2}$

102. $y = e^{-2x} + 2, y = 0, x = 0, x = 2$

103. Given the exponential function $f(x) = e^x$, show that

(a) $f(u - v) = \frac{f(u)}{f(v)}$. (b) $f(kx) = [f(x)]^k$.

104. Approximate each integral using the Midpoint Rule, the Trapezoidal Rule, and Simpson's Rule with $n = 12$. Then use the integration capabilities of a graphing utility to approximate the integrals and compare the results.

(a) $\int_0^4 \sqrt{x}\, e^x\, dx$ (b) $\int_0^2 2xe^{-x}\, dx$

105. ***Probability*** A car battery has an average lifetime of 48 months with a standard deviation of 6 months. The battery lives are normally distributed. The probability that a given battery will last between 48 months and 60 months is

$$0.0665 \int_{48}^{60} e^{-0.0139(t-48)^2}\, dt.$$

Use the integration capabilities of a graphing utility to approximate the integral. Interpret the resulting probability.

106. Given $e^x \geq 1$ for $x \geq 0$, it follows that

$$\int_0^x e^t\, dt \geq \int_0^x 1\, dt.$$

Perform this integration to derive the inequality $e^x \geq 1 + x$ for $x \geq 0$.

107. ***Modeling Data*** A valve on a storage tank is opened for 4 hours to release a chemical in a manufacturing process. The flow rate R (in liters per hour) at time t (in hours) is given in the table.

t	0	1	2	3	4
R	425	240	118	71	36

(a) Use the regression capabilities of a graphing utility to find a linear model for the points $(t, \ln R)$. Write the resulting equation of the form $\ln R = at + b$ in exponential form.

(b) Use a graphing utility to plot the data and graph the exponential model.

(c) Use the definite integral to approximate the number of liters of chemical released during the 4 hours.

108. Prove that $\frac{e^a}{e^b} = e^{a-b}$.

109. Let $f(x) = \frac{\ln x}{x}$.

(a) Graph f on $(0, \infty)$ and show that f is strictly decreasing on $[e, \infty)$.

(b) Show that if $e \leq A < B$, then $A^B > B^A$.

(c) Use part (b) to show that $e^\pi > \pi^e$.

EXERCISES FOR SECTION 5.5

In Exercises 1–4, evaluate the expression without using a calculator.

1. $\log_2 \left(\frac{1}{8}\right)$
2. $\log_{27} 9$
3. $\log_7 1$
4. $\log_a \frac{1}{a}$

In Exercises 5–8, write the exponential equation as a logarithmic equation or vice versa.

5. (a) $2^3 = 8$
   (b) $3^{-1} = \frac{1}{3}$
6. (a) $27^{2/3} = 9$
   (b) $16^{3/4} = 8$
7. (a) $\log_{10} 0.01 = -2$
   (b) $\log_{0.5} 8 = -3$
8. (a) $\log_3 \frac{1}{9} = -2$
   (b) $49^{1/2} = 7$

In Exercises 9–14, solve for x or b.

9. (a) $\log_{10} 1000 = x$
   (b) $\log_{10} 0.1 = x$
10. (a) $\log_4 \frac{1}{64} = x$
    (b) $\log_5 25 = x$
11. (a) $\log_3 x = -1$
    (b) $\log_2 x = -4$
12. (a) $\log_b 27 = 3$
    (b) $\log_b 125 = 3$
13. (a) $x^2 - x = \log_5 25$
    (b) $3x + 5 = \log_2 64$
14. (a) $\log_3 x + \log_3(x - 2) = 1$
    (b) $\log_{10}(x + 3) - \log_{10} x = 1$

In Exercises 15–20, sketch the graph of the function by hand.

15. $y = 3^x$
16. $y = 3^{x-1}$
17. $y = \left(\frac{1}{3}\right)^x$
18. $y = 2^{x^2}$
19. $h(x) = 5^{x-2}$
20. $y = 3^{-|x|}$

In Exercises 21–24, use a graphing utility to graph the function and approximate its zero(s) accurate to three decimal places.

21. $g(x) = 6(2^{1-x}) - 25$
22. $f(t) = 300(1.0075^{12t}) - 735.41$
23. $h(s) = 32 \log_{10}(s - 2) + 15$
24. $g(x) = 1 - 2 \log_{10}[x(x - 3)]$

In Exercises 25 and 26, illustrate that the functions are inverses of each other by sketching their graphs on the same set of coordinate axes.

25. $f(x) = 4^x$
    $g(x) = \log_4 x$
26. $f(x) = 3^x$
    $g(x) = \log_3 x$

27. ***Think About It*** The table of values was obtained from evaluating a function. Determine which of the statements may be true and which must be false, and explain why.
    (a) y is an exponential function of x.
    (b) y is a logarithmic function of x.
    (c) x is an exponential function of y.
    (d) y is a linear function of x.

x	1	2	8
y	0	1	3

28. ***Think About It*** Consider the function $f(x) = \log_{10} x$.
    (a) What is the domain of f?
    (b) Find f^{-1}.
    (c) If x is a real number between 1000 and 10,000, determine the interval in which $f(x)$ will be found.
    (d) Determine the interval in which x will be found if $f(x)$ is negative.
    (e) If $f(x)$ is increased by one unit, x must have been increased by what factor?
    (f) Find the ratio of x_1 to x_2 given that $f(x_1) = 3n$ and $f(x_2) = n$.

In Exercises 29–44, find the derivative of the function.

29. $f(x) = 4^x$
30. $g(x) = 2^{-x}$
31. $y = 5^{x-2}$
32. $y = x(7^{-3x})$
33. $g(t) = t^2 2^t$
34. $f(t) = \dfrac{3^{2t}}{t}$
35. $h(\theta) = 2^{-\theta} \cos \pi\theta$
36. $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$
37. $y = \log_3 x$
38. $y = \log_{10} 2x$
39. $f(x) = \log_2 \dfrac{x^2}{x - 1}$
40. $h(x) = \log_3 \dfrac{x\sqrt{x - 1}}{2}$
41. $y = \log_5 \sqrt{x^2 - 1}$
42. $y = \log_{10} \dfrac{x^2 - 1}{x}$
43. $g(t) = \dfrac{10 \log_4 t}{t}$
44. $f(t) = t^{3/2} \log_2 \sqrt{t + 1}$

In Exercises 45–48, use logarithmic differentiation to find dy/dx.

45. $y = x^{2/x}$
46. $y = x^{x-1}$
47. $y = (x - 2)^{x+1}$
48. $y = (1 + x)^{1/x}$

49. ***Ordering Functions*** Order the functions $f(x) = \log_2 x$, $g(x) = x^x$, $h(x) = x^2$, and $k(x) = 2^x$ from the one with the greatest rate of growth to the one with the smallest rate of growth for "large" values of x.

50. Given the exponential function $f(x) = a^x$, show that

(a) $f(u + v) = f(u) \cdot f(v)$.

(b) $f(2x) = [f(x)]^2$.

51. ***Inflation*** If the annual rate of inflation averages 5% over the next 10 years, the approximate cost C of goods or services during any year in that decade is

$$C(t) = P(1.05)^t$$

where t is the time in years and P is the present cost.

(a) If the price of an oil change for your car is presently \$24.95, estimate the price 10 years from now.

(b) Find the rate of change of C with respect to t when $t = 1$ and $t = 8$.

(c) Verify that the rate of change of C is proportional to C. What is the constant of proportionality?

52. ***Depreciation*** After t years, the value of a car purchased for \$20,000 is

$$V(t) = 20{,}000\left(\tfrac{3}{4}\right)^t.$$

(a) Use a graphing utility to graph the function and determine the value of the car 2 years after it was purchased.

(b) Find the rate of change of V with respect to t when $t = 1$ and $t = 4$.

(c) Use a graphing utility to graph $V'(t)$ and determine the horizontal asymptote of $V'(t)$. Interpret its meaning in the context of the problem.

Compound Interest **In Exercises 53–56, complete the table to determine the balance A for P dollars invested at rate r for t years and compounded n times per year.**

n	1	2	4	12	365	Continuous compounding
A						

53. $P = \$1000$, $r = 3\frac{1}{2}\%$, $t = 10$ years

54. $P = \$2500$, $r = 6\%$, $t = 20$ years

55. $P = \$1000$, $r = 5\%$, $t = 30$ years

56. $P = \$2500$, $r = 5\%$, $t = 40$ years

Compound Interest **In Exercises 57–60, complete the table to determine the amount of money P (present value) that should be invested at rate r to produce a balance of \$100,000 in t years.**

t	1	10	20	30	40	50
P						

57. $r = 5\%$, Compounded continuously

58. $r = 6\%$, Compounded continuously

59. $r = 5\%$, Compounded monthly

60. $r = 7\%$, Compounded daily

61. ***Compound Interest*** Assume that you can earn 6% on an investment, compounded daily. Which of the following options would yield the greatest balance in 8 years?

(a) \$20,000 now

(b) \$30,000 in 8 years

(c) \$8000 now and \$20,000 in 4 years

(d) \$9000 now, \$9000 in 4 years, and \$9000 in 8 years

62. ***Compound Interest*** Consider a deposit of \$100 placed in an account for 20 years at r% compounded continuously. Use a graphing utility to graph the exponential functions giving the growth of the investment over the 20 years for each of the following interest rates. Compare the ending balances for each of the rates.

(a) $r = 3\%$ (b) $r = 5\%$ (c) $r = 6\%$

63. ***Timber Yield*** The yield V (in millions of cubic feet per acre) for a stand of timber at age t is

$$V = 6.7e^{(-48.1)/t}$$

where t is measured in years.

(a) Find the limiting volume of wood per acre as t approaches infinity.

(b) Find the rate at which the yield is changing when $t = 20$ years and $t = 60$ years.

64. ***Learning Theory*** In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be

$$P = \frac{0.83}{1 + e^{-0.2n}}.$$

(a) Find the limiting proportion of correct responses as n approaches infinity.

(b) Find the rate at which P is changing after $n = 3$ trials and $n = 10$ trials.

65. ***Forest Defoliation*** To estimate the amount of defoliation caused by the gypsy moth during a year, a forester counts the number of egg masses on $\frac{1}{40}$ of an acre the preceding fall. The percent of defoliation y is approximated by

$$y = \frac{300}{3 + 17e^{-0.0625x}}$$

where x is the number of egg masses in thousands. *(Source: USDA Forest Service)*

(a) Use a graphing utility to graph the function.

(b) Estimate the percent of defoliation if 2000 egg masses are counted.

(c) Estimate the number of egg masses that existed if you observe that approximately $\frac{2}{3}$ of a forest is defoliated.

(d) Use calculus to estimate the value of x for which y is increasing most rapidly.

66. ***Population Growth*** A lake is stocked with 500 fish, and their population increases according to the logistics curve

$$p(t) = \frac{10{,}000}{1 + 19e^{-t/5}}$$

where t is measured in months.

(a) Use a graphing utility to graph the function.

(b) What is the limiting size of the fish population?

(c) At what rates is the fish population changing at the end of 1 month and at the end of 10 months?

(d) After how many months is the population increasing most rapidly?

67. ***Modeling Data*** The table gives the national health expenditures y (in billions of dollars) for the years 1984 through 1993, with $x = 4$ corresponding to 1984. *(Source: U.S. Health Care Financing Administration)*

x	4	5	6	7	8
y	396.0	434.5	466.0	506.2	562.3

x	9	10	11	12	13
y	623.9	696.6	755.6	820.3	884.2

(a) Use a graphing utility to find an exponential model for the data. Use the graphing utility to plot the data and graph the exponential model.

(b) Use a graphing utility to find a logarithmic model for the data. Use the graphing utility to plot the data and graph the logarithmic model.

(c) Which model better approximates the data?

(d) Find the rate of growth of each model for the year 2000. If the rate of growth of health care expenditures could be slowed, which may be the better model for the future? Explain.

68. ***Comparing Models*** The amount y (in billions of dollars) given to philanthropy (from individuals, foundations, corporations, and charitable bequests) for the years 1983 through 1993 in the United States is given in the table, with $x = 3$ corresponding to 1983. *(Source: AAFRC Trust for Philanthropy)*

x	3	4	5	6	7	8
y	63.2	68.8	73.2	83.9	90.3	98.4

x	9	10	11	12	13
y	107.0	111.9	117.1	121.9	126.2

(a) Use the regression capabilities of a graphing utility to find the following models for the data.

$y_1 = ax + b$ $\quad$ $y_2 = a + b \ln x$

$y_3 = ab^x$ $\quad$ $y_4 = ax^b$

(b) Use a graphing utility to plot the data and graph each model. Which model do you think best fits the data?

(c) Interpret the slope of the linear model in the context of the problem.

(d) Find the rate of change of each model for the year 1992. Which model is increasing at the greatest rate in 1992?

In Exercises 69–76, evaluate the integral.

69. $\int 3^x\,dx$ $\qquad$ **70.** $\int 4^{-x}\,dx$

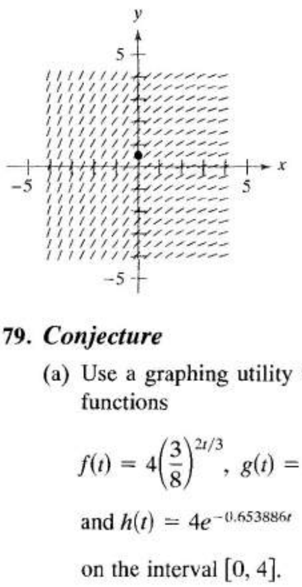
71. $\int_{-1}^{2} 2^x\,dx$ $\qquad$ **72.** $\int_{-2}^{0} (3^3 - 5^2)\,dx$

73. $\int x(5^{-x^2})\,dx$ $\qquad$ **74.** $\int (3 - x)7^{(3-x)^2}\,dx$

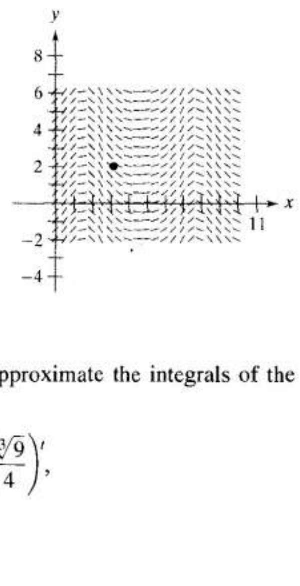
75. $\int \frac{3^{2x}}{1 + 3^{2x}}\,dx$ $\qquad$ **76.** $\int 2^{\sin x}\cos x\,dx$

Direction Fields **In Exercises 77 and 78, a differential equation, a point, and a direction field are given. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

77. $\frac{dy}{dx} = 0.4^{x/3}, \quad \left(0, \frac{1}{2}\right)$



78. $\frac{dy}{dx} = e^{\sin x}\cos x, \quad (\pi, 2)$



79. ***Conjecture***

(a) Use a graphing utility to approximate the integrals of the functions

$$f(t) = 4\left(\frac{3}{8}\right)^{2t/3}, \quad g(t) = 4\left(\frac{\sqrt[3]{9}}{4}\right)^t,$$

and $h(t) = 4e^{-0.653886t}$

on the interval $[0, 4]$.

(b) Use a graphing utility to graph the three functions.

(c) Use the results in parts (a) and (b) to make a conjecture about the three functions. Could you make the conjecture using only part (a)? Explain. Prove your conjecture analytically.

80. ***Area*** Find the area of the region bounded by the graphs of $y = 3^x$, $y = 0$, $x = 0$, and $x = 3$.

81. ***Continuous Cash Flow*** The present value P of a continuous cash flow of \$2000 per year earning 6% compounded continuously over 10 years is

$$P = \int_0^{10} 2000e^{-0.06t}\,dt.$$

Find P.

82. Complete the table to demonstrate that e can also be defined as $\lim_{x\to 0^+} (1 + x)^{1/x}$.

x	1	10^{-1}	10^{-2}	10^{-4}	10^{-6}
$(1 + x)^{1/x}$					

In Exercises 83 and 84, find an exponential function that fits the experimental data collected over time *t*.

83.

t	0	1	2	3	4
y	1200.00	720.00	432.00	259.20	155.52

84.

t	0	1	2	3	4
y	600.00	630.00	661.50	694.58	729.30

True or False? **In Exercises 85–90, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

85. $e = 271{,}801/99{,}990$.

86. If $f(x) = \ln x$, then $f(e^{n+1}) - f(e^n) = 1$ for any value of n.

87. The functions $f(x) = 2 + e^x$ and $g(x) = \ln(x - 2)$ are inverses of each other.

88. The exponential function $y = Ce^x$ is a solution of the differential equation $d^n y/dx^n = y$, $n = 1, 2, 3, \ldots$.

89. The graphs of $f(x) = e^x$ and $g(x) = e^{-x}$ meet at right angles.

90. If $f(x) = g(x)e^x$, then the only zeros of f are the zeros of g.

91. Solve the logistics differential equation

$$\frac{dy}{dt} = \frac{8}{25}y\left(\frac{5}{4} - y\right), \quad y(0) = 1$$

and obtain the logistics growth function of Example 7.

$$\left[\text{Hint: } \frac{1}{y\left(\frac{5}{4} - y\right)} = \frac{4}{5}\left(\frac{1}{y} + \frac{1}{\frac{5}{4} - y}\right)\right]$$

92. Find an equation of the tangent line to $y = x^{\sin x}$ at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

SECTION PROJECT

Let $f(x) = \begin{cases} |x|^x, & x \neq 0 \\ 1, & x = 0. \end{cases}$

(a) Use a graphing utility to graph f in the viewing rectangle $-3 \le x \le 3$, $-2 \le y \le 2$. What is the domain of f?

(b) Use the zoom and trace features of a graphing utility to estimate

$$\lim_{x\to 0} f(x).$$

(c) Write a short paragraph explaining why the function f is continuous for all real numbers.

(d) Visually estimate the slope of f at the point $(0, 1)$.

(e) Explain why the derivative of a function can be approximated by the formula

$$\frac{f(x + h) - f(x - h)}{2h}$$

for small values of h. Use this formula to approximate the slope of f at the point $(0, 1)$.

$$f'(0) \approx \frac{f(0 + h) - f(0 - h)}{2h} = \frac{f(h) - f(-h)}{2h}$$

What do you think the slope of the graph of f is at $(0, 1)$?

(f) Find a formula for the derivative of f and determine $f'(0)$. Write a short paragraph explaining how a graphing utility might lead you to approximate the slope of a graph incorrectly.

(g) Use your formula for the derivative of f to find the relative extrema of f. Verify your answer with a graphing utility.

FOR FURTHER INFORMATION For more information on using graphing utilities to estimate slope, see the article "Computer-Aided Delusions" by Richard L. Hall in the September 1993 issue of *The College Mathematics Journal*.

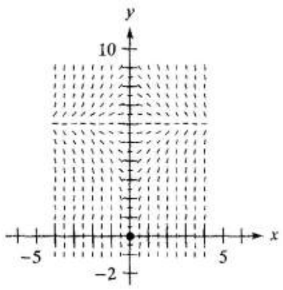
EXERCISES FOR SECTION 5.6

In Exercises 1–6, solve the differential equation.

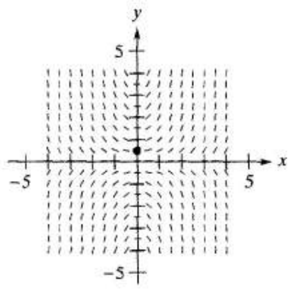
1. $y' = \dfrac{5x}{y}$
2. $y' = \dfrac{\sqrt{x}}{2y}$
3. $y' = \sqrt{x}\,y$
4. $y' = x(1 + y)$
5. $(1 + x^2)y' - 2xy = 0$
6. $xy + y' = 100x$

Direction Fields **In Exercises 7 and 8, a differential equation, a point, and a direction field are given. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketch in part (a).**

7. $\dfrac{dy}{dx} = x(6 - y)$, $(0, 0)$



8. $\dfrac{dy}{dx} = xy$, $\left(0, \frac{1}{2}\right)$



Think About It **In Exercises 9–12, write and solve the differential equation that models the verbal statement.**

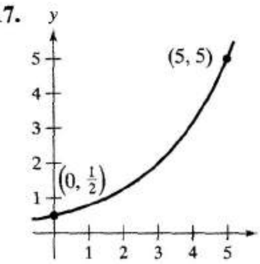
9. The rate of change of Q with respect to t is inversely proportional to the square of t.
10. The rate of change of P with respect to t is proportional to $10 - t$.
11. The rate of change of N with respect to s in proportional to $250 - s$.
12. The rate of change of y with respect to x varies jointly as x and $L - y$.

In Exercises 13–16, find the function $y = f(t)$ passing through the point $(0, 10)$ with the given first derivative. Use a graphing utility to graph the solution.

13. $\dfrac{dy}{dt} = \dfrac{1}{2}t$
14. $\dfrac{dy}{dt} = -\dfrac{3}{4}\sqrt{t}$
15. $\dfrac{dy}{dt} = -\dfrac{1}{2}y$
16. $\dfrac{dy}{dt} = \dfrac{3}{4}y$

In Exercises 17–20, find the exponential function $y = Ce^{kt}$ that passes through the two given points.

17.



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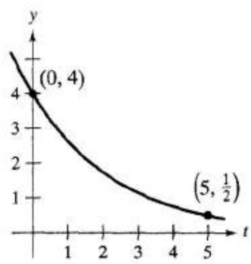

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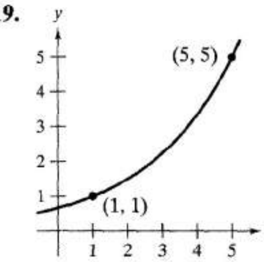

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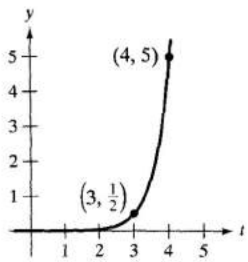

Radioactive Decay **In Exercises 21–26, complete the table for the radioactive isotope.**

	Isotope	Half-Life (in years)	Initial Quantity	Amount After 1000 years	Amount After 10,000 years
21.	Ra^{226}	1620	10g		
22.	Ra^{226}	1620		1.5g	
23.	C^{14}	5730			2g
24.	C^{14}	5730	3g		
25.	Pu^{239}	24,360		2.1g	
26.	Pu^{239}	24,360			0.4g

27. ***Radioactive Decay*** Radioactive radium has a half-life of approximately 1620 years. What percent of a given amount remains after 100 years?

28. ***Carbon Dating*** Carbon-14 dating assumes that the carbon dioxide on earth today has the same radioactive content as it did centuries ago. If this is true, the amount of C^{14} absorbed by a tree that grew several centuries ago should be the same as the amount of C^{14} absorbed by a tree growing today. A piece of ancient charcoal contains only 15% as much of the radioactive carbon as a piece of modern charcoal. How long ago was the tree burned to make the ancient charcoal? (The half-life of C^{14} is 5730 years.)

Compound Interest **In Exercises 29–34, complete the table for a savings account in which interest is compounded continuously.**

	Initial Investment	*Annual Rate*	*Time to Double*	*Amount After 10 Years*
29.	\$1000	6%		
30.	\$20,000	$5\frac{1}{2}\%$		
31.	\$750		$7\frac{3}{4}$ yr	
32.	\$10,000		5 yr	
33.	\$500			\$1292.85
34.	\$2000			\$5436.56

Compound Interest **In Exercises 35 and 36, find the principal P that must be invested at rate r, compounded monthly, so that \$500,000 will be available for retirement in t years.**

35. $r = 7\frac{1}{2}\%, \quad t = 20$

36. $r = 6\%, \quad t = 40$

Compound Interest **In Exercises 37 and 38, find the time necessary for \$1000 to double if it is invested at a rate of r compounded (a) annually, (b) monthly, (c) daily, and (d) continuously.**

37. $r = 7\%$

38. $r = 6\frac{1}{2}\%$

Population **In Exercises 39–42, the table gives the population (in millions) of a city in 1990 and the projected population (in millions) for the year 2000. Find the exponential growth model $y = Ce^{kt}$ for the population by letting $t = 0$ correspond to 1990. Use the model to predict the population of the city in 2010.**

	City	*1900*	*2000*
39.	Dhaka, Bangladesh	4.22	6.49
40.	Houston, Texas	2.30	2.65
41.	Detroit, Michigan	3.00	2.74
42.	London, United Kingdom	9.17	8.57

43. ***Think About It***

(a) In Exercises 39–42, what constant in the equation $y = Ce^{kt}$ represents the growth rate? Write a sentence that describes the growth rate in common English.

(b) In Exercises 39 and 41, you can see that one population is increasing whereas the other is decreasing. What constant in the equation $y = Ce^{kt}$ reflects this difference? Explain.

44. ***Modeling Data*** One hundred bacteria are started in a culture and the number N of bacteria is counted each hour for 5 hours. The results are given in the table, where t is the time in hours.

t	0	1	2	3	4	5
N	100	126	151	198	243	297

(a) Use the regression capabilities of a graphing utility to find an exponential model for the data.

(b) Use the model to estimate the time required for the population to quadruple in size.

45. ***Atmospheric Pressure*** Atmospheric pressure P (measured in millimeters of mercury) decreases exponentially with increasing altitude x (measured in meters). The pressure is 760 millimeters of mercury at sea level ($x = 0$) and 672.71 millimeters of mercury at an altitude of 1000 meters. Find the pressure at an altitude of 3000 meters.

46. ***Revenue*** Because of a slump in the economy, a company finds that its annual revenues have dropped from \$742,000 in 1996 to \$632,000 in 1998. If the revenue is following an exponential pattern of decline, what is the expected revenue for 1999? (Let $t = 0$ represent 1996.)

47. ***Learning Curve*** The management at a certain factory has found that a worker can produce at most 30 units in a day. The learning curve for the number of units N produced per day after a new employee has worked t days is

$$N = 30(1 - e^{kt}).$$

After 20 days on the job, a particular worker produces 19 units.

(a) Find the learning curve for this worker.

(b) How many days should pass before this worker is producing 25 units per day?

48. ***Learning Curve*** If in Exercise 47 management requires a new employee to produce at least 20 units per day after 30 days on the job, find (a) the learning curve that describes this minimum requirement and (b) the number of days before a minimal achiever is producing 25 units per day.

49. ***Sales*** The sales S (in thousands of units) of a new product after it has been on the market for t years is

$$S = Ce^{k/t}.$$

(a) Find S as a function of t if 5000 units have been sold after 1 year and the saturation point for the market is 30,000 (that is, $\lim_{t \to \infty} S = 30$).

(b) How many units will have been sold after 5 years?

(c) Use a graphing utility to graph this sales function.

50. ***Sales*** The sales S (in thousands of units) of a new product after it has been on the market for t years is

$$S = 30(1 - e^{kt}).$$

(a) Find S as a function of t if 5000 units have been sold after 1 year.

(b) How many units will saturate this market?

(c) How many units will have been sold after 5 years?

(d) Use a graphing utility to graph this sales function.

51. ***Forestry*** The value of a tract of timber is

$$V(t) = 100{,}000e^{0.8\sqrt{t}}$$

where t is the time in years, with $t = 0$ corresponding to 1998. If money earns interest continuously at 10%, the present value of the timber at any time t is

$$A(t) = V(t)e^{-0.10t}.$$

Find the year in which the timber should be harvested to maximize the present value function.

52. ***Modeling Data*** The table gives the net receipts and the amounts required to service the national debt of the United States from 1986 through 1995. The monetary amounts are given in billions of dollars. *(Source: U.S. Office of Management and Budget)*

Year	1986	1987	1988	1989	1990
Receipts	769.1	854.1	909.0	990.7	1031.3
Interest	136.0	138.7	151.8	169.3	184.2

Year	1991	1992	1993	1994	1995
Receipts	1054.3	1090.5	1153.5	1257.7	1346.4
Interest	194.5	199.4	198.8	203.0	234.2

(a) Use the regression capabilities of a graphing utility to find an exponential model R for the receipts and a logarithmic model I for the amount required to service the debt. Let t represent the time in years, with $t = 0$ corresponding to 1980.

(b) Use a graphing utility to plot the receipt data and graph the exponential model. Based on the model, what is the continuous rate of growth of the receipts?

(c) Use a graphing utility to plot the interest data and graph the logarithmic model.

(d) Find a function $P(t)$ that approximates the percent of the receipts that is required to service the national debt. Use a graphing utility to graph this function.

53. ***Sound Intensity*** The level of sound β, in decibels, with an intensity of I is

$$\beta(I) = 10 \log_{10} \frac{I}{I_0}$$

where I_0 is an intensity of 10^{-16} watts per square centimeter, corresponding roughly to the faintest sound that can be heard. Determine $\beta(I)$ for the following.

(a) $I = 10^{-14}$ watts per square centimeter (whisper)

(b) $I = 10^{-9}$ watts per square centimeter (busy street corner)

(c) $I = 10^{-6.5}$ watts per square centimeter (air hammer)

(d) $I = 10^{-4}$ watts per square centimeter (threshold of pain)

54. ***Noise Level*** With the installation of noise suppression materials, the noise level in an auditorium was reduced from 93 to 80 decibels. Use the function in Exercise 53 to find the percent decrease in the intensity level of the noise as a result of the installation of these materials.

55. ***Earthquake Intensity*** On the Richter scale, the magnitude R of an earthquake of intensity I is

$$R = \frac{\ln I - \ln I_0}{\ln 10}$$

where I_0 is the minimum intensity used for comparison. Assume that $I_0 = 1$.

(a) Find the intensity of the 1906 San Francisco earthquake ($R = 8.3$).

(b) Find the factor by which the intensity is increased if the Richter scale measurement is doubled.

(c) Find dR/dI.

56. ***Newton's Law of Cooling*** When an object is removed from a furnace and placed in an environment with a constant temperature of 90°F, its core temperature is 1500°F. One hour after it is removed, the core temperature is 1120°F. Find the core temperature 5 hours after the object is removed from the furnace.

57. ***Newton's Law of Cooling*** A thermometer is taken from a room at 72°F to the outdoors where the temperature is 20°F. Determine the reading on the thermometer after 5 minutes, if the reading drops to 48°F after 1 minute.

58. ***Comparing Models*** The time t (in seconds) required to attain a speed of s miles per hour from a standing start for a 1995 Dodge Avenger is given in the table. *(Source: Road & Track, March 1995)*

s	30	40	50	60	70	80	90
t	3.4	5.0	7.0	9.3	12.0	15.8	20.0

(a) Use a graphing utility to find an exponential model for the data.

(b) Use a graphing utility to plot the data and graph the model.

(c) Find t and dt/ds for $s = 55$.

59. ***Home Mortgage*** A \$120,000 home mortgage for 35 years at $9\frac{1}{2}\%$ has a monthly payment of \$985.93. Part of the monthly payment goes for the interest charge on the unpaid balance and the remainder of the payment is used to reduce the principal. The amount that goes for interest is

$$u = M - \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}$$

and the amount that goes toward reduction of the principal is

$$v = \left(M - \frac{Pr}{12}\right)\left(1 + \frac{r}{12}\right)^{12t}.$$

In these formulas, P is the size of the mortgage, r is the interest rate, M is the monthly payment, and t is the time in years.

(a) Use a graphing utility to graph each function in the same viewing rectangle. (The viewing rectangle should show all 35 years of mortgage payments.)

(b) In the early years of the mortgage, the larger part of the monthly payment goes for what purpose? Approximate the time when the monthly payment is evenly divided between interest and principal reduction.

(c) Use the graphs to make a conjecture about the relationship between the slopes of the tangent lines to the two curves for a specified value of t. Give an analytical argument to verify your conjecture. Find $u'(15)$ and $v'(15)$.

(d) Repeat parts (a) and (b) for a repayment period of 20 years ($M = \$1118.56$). What can you conclude?