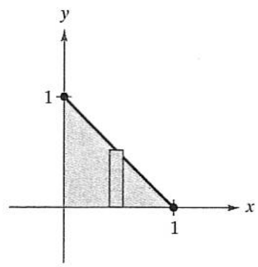
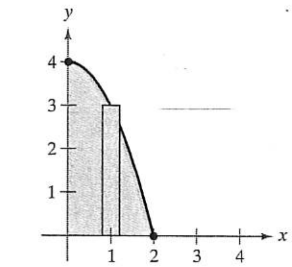
EXERCISES FOR SECTION 6.2

In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

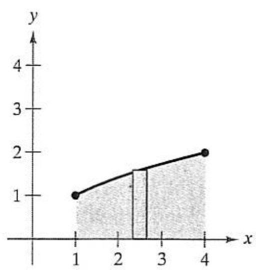
1. $y = -x + 1$



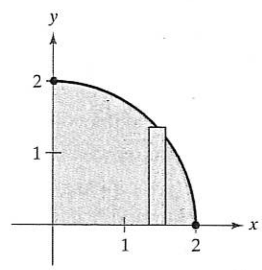
2. $y = 4 - x^2$



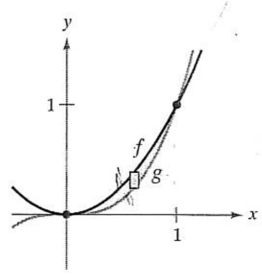
3. $y = \sqrt{x}$



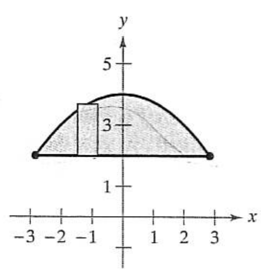
4. $y = \sqrt{4 - x^2}$



5. $y = x^2,\ y = x^3$

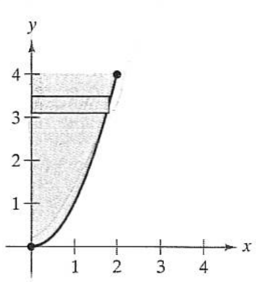


6. $y = 2,\ y = 4 - \dfrac{x^2}{4}$

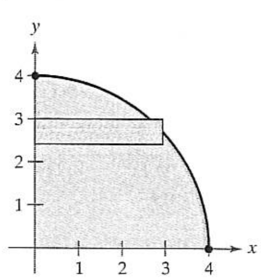


In Exercises 7–10, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

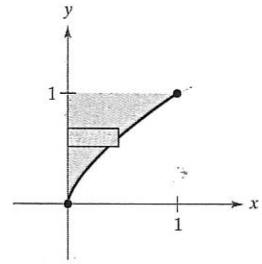
7. $y = x^2$



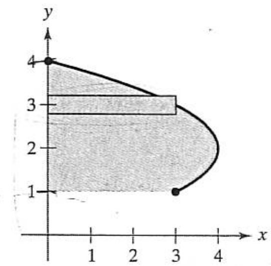
8. $y = \sqrt{16 - x^2}$



9. $y = x^{2/3}$



10. $x = -y^2 + 4y$



In Exercises 11–14, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated lines.

11. $y = \sqrt{x},\ y = 0,\ x = 4$
    (a) the x-axis (b) the y-axis
    (c) the line $x = 4$ (d) the line $x = 6$

12. $y = 2x^2,\ y = 0,\ x = 2$
    (a) the y-axis (b) the x-axis
    (c) the line $y = 8$ (d) the line $x = 2$

13. $y = x^2,\ y = 4x - x^2$
    (a) the x-axis (b) the line $y = 6$

14. $y = 6 - 2x - x^2,\ y = x + 6$
    (a) the x-axis (b) the line $y = 3$

In Exercises 15–18, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $y = 4$.

15. $y = x,\ y = 3,\ x = 0$

16. $y = x^2,\ y = 4$

17. $y = \dfrac{1}{x},\ y = 0,\ x = 1,\ x = 4$

18. $y = \sec x,\ y = 0,\ 0 \le x \le \dfrac{\pi}{3}$

In Exercises 19–22, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line $x = 6$.

19. $y = x,\ y = 0,\ y = 4,\ x = 6$

20. $y = 6 - x,\ y = 0,\ y = 4,\ x = 0$

21. $x = y^2,\ x = 4$

22. $xy = 6,\ y = 2,\ y = 6,\ x = 6$

In Exercises 23–28, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

23. $y = \dfrac{1}{\sqrt{x+1}}, \quad y = 0, \quad x = 0, \quad x = 3$

24. $y = x\sqrt{4 - x^2}, \quad y = 0$

25. $y = \dfrac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 4$

26. $y = \dfrac{3}{x+1}, \quad y = 0, \quad x = 0, \quad x = 8$

27. $y = e^{-x}, \quad y = 0, \quad x = 0, \quad x = 1$

28. $y = e^{x/2}, y = 0, \quad x = 0, \quad x = 4$

In Exercises 29 and 30, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y-axis.

29. $y = 3(2 - x), \quad y = 0, \quad x = 0$

30. $y = 9 - x^2, \quad y = 0, \quad x = 2, \quad x = 3$

In Exercises 31–36, use the integration capabilities of a graphing utility to approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

31. $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$

32. $y = \cos x, \quad y = 0, \quad x = 0, \quad x = \dfrac{\pi}{2}$

33. $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 2$

34. $y = \ln x, \quad y = 0, \quad x = 1, \quad x = 3$

35. $y = e^{x/2} + e^{-x/2}, \quad y = 0, \quad x = -1, \quad x = 2$

36. $y = 2\arctan(0.2x), \quad y = 0, \quad x = 0, \quad x = 5$

Approximation **In Exercises 37 and 38, determine which value best approximates the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis. (Make your selection on the basis of a sketch of the solid and *not* by performing any calculations.)**

37. $y = e^{-x^2/2}, \quad y = 0, \quad x = 0, \quad x = 2$

(a) 3 (b) -5 (c) 10 (d) 7 (e) 20

38. $y = \arctan x, \quad y = 0, \quad x = 0, \quad x = 1$

(a) 10 (b) $\frac{3}{4}$ (c) 5 (d) -6 (e) 15

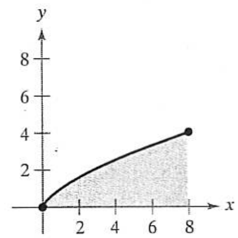
39. ***Think About It***

(a) The region bounded by the parabola $y = 4x - x^2$ and the x-axis is revolved about the x-axis. Find the volume of the resulting solid.

(b) If the equation of the parabola in part (a) were changed to $y = 4 - x^2$, would the volume of the solid generated be different? Explain.

40. ***Think About It*** The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.

(a) x-axis (b) y-axis (c) $x = 8$



41. If the portion of the line $y = \frac{1}{2}x$ lying in the first quadrant is revolved about the x-axis, a cone is generated. Find the volume of the cone extending from $x = 0$ to $x = 6$.

42. Use the disc method to verify that the volume of a right circular cone is $\frac{1}{3}\pi r^2 h$, where r is the radius of the base and h is the height.

43. Use the disc method to verify that the volume of a sphere is $\frac{4}{3}\pi r^3$.

44. A sphere of radius r is cut by a plane h $(h < r)$ units above the equator. Find the volume of the solid (spherical segment) above the plane.

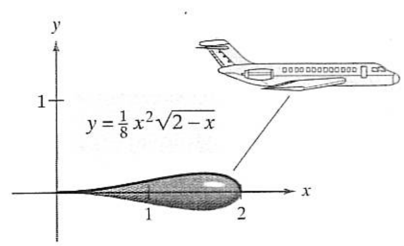
45. A cone with a base of radius r and height H is cut by a plane parallel to and h units above the base. Find the volume of the solid (frustum of a cone) below the plane.

46. The region bounded by $y = \sqrt{x}$, $y = 0$, $x = 0$, and $x = 4$ is revolved about the x-axis.

(a) Find the value of x in the interval $[0, 4]$ that divides the solid into two parts of equal volume.

(b) Find the values of x in the interval $[0, 4]$ that divide the solid into three parts of equal volume.

47. ***Volume of a Fuel Tank*** A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2 - x}$ and the x-axis about the x-axis (see figure), where x and y are measured in meters. Find the tank's volume.



48. ***Volume of a Lab Glass*** A glass container can be modeled by revolving the graph of

$$y = \begin{cases} \sqrt{0.1x^3 - 2.2x^2 + 10.9x + 22.2}, & 0 \le x \le 11.5 \\ 2.95, & 11.5 < x \le 15 \end{cases}$$

about the x-axis, where x and y are measured in centimeters. Use a graphing utility to graph the function and find the volume of the container.

49. Find the volume of the solid generated if the upper half of the ellipse $9x^2 + 25y^2 = 225$ is revolved about

(a) the x-axis to form a prolate spheroid (shaped like a football).

(b) the y-axis to form an oblate spheroid (shaped like half of an M&M candy).

50. ***Minimum Volume*** The arc of $y = 4 - (x^2/4)$ on the interval $[0, 4]$ is revolved about the line $y = b$ (see figure).

(a) Find the volume of the resulting solid as a function of b.

(b) Use a graphing utility to graph the function in part (a), and use the graph to approximate the value of b that minimizes the volume of the solid.

(c) Use calculus to find the value of b that minimizes the volume of the solid, and compare the result with the answer to part (b).

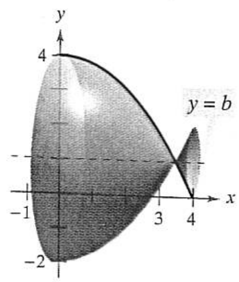


Figure for 50

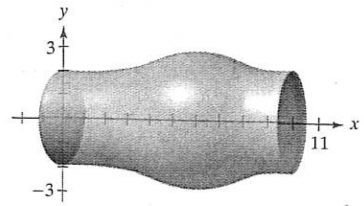


Figure for 52

51. ***Water Depth in a Tank*** A tank on a water tower is a sphere of radius 50 feet. Determine the depths of the water when the tank is filled to one-fourth and three-fourths of its total capacity. (*Note:* Use the root-finding capabilities of a graphing utility after evaluating the definite integral.)

52. ***Modeling Data*** A draftsman is asked to determine the amount of material required to produce a machine part (see figure). The diameters d of the part at equally spaced points x are listed in the table. The measurements are listed in centimeters.

x	0	1	2	3	4	5
d	4.2	3.8	4.2	4.7	5.2	5.7

x	6	7	8	9	10
d	5.8	5.4	4.9	4.4	4.6

(a) Use these data with Simpson's Rule to approximate the volume of the part.

(b) Use the regression capabilities of a graphing utility to find a fourth-degree polynomial through the points representing the radius of the solid. Plot the data and graph the model.

(c) Use a graphing utility to approximate the definite integral yielding the volume of the part. Compare the result with the answer to part (a).

53. ***Think About It*** Match each integral with the solid whose volume it represents, and give the dimensions of each solid.

(a) Right circular cylinder (b) Ellipsoid

(c) Sphere (d) Right circular cone (e) Torus

(i) $\pi \int_0^h \left(\frac{rx}{h}\right)^2 dx$

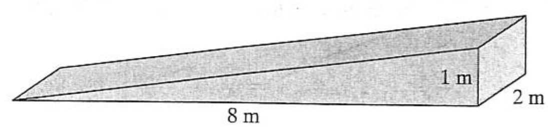
(ii) $\pi \int_0^h r^2\, dx$

(iii) $\pi \int_{-r}^{r} \left(\sqrt{r^2 - x^2}\right)^2 dx$

(iv) $\pi \int_{-b}^{b} \left(\sqrt{1 - \frac{x^2}{b^2}}\right)^2 dx$

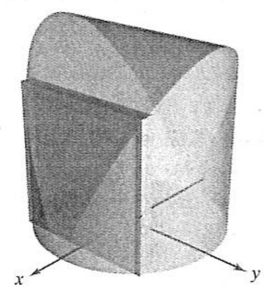
(v) $\pi \int_{-r}^{r} \left[\left(R + \sqrt{r^2 - x^2}\right)^2 - \left(R - \sqrt{r^2 - x^2}\right)^2\right] dx$

54. Find the volume of concrete in a ramp that is 2 meters wide and whose cross sections are right triangles with base 8 meters and height 1 meter (see figure).

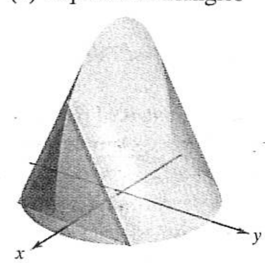


55. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$, with the indicated cross sections taken perpendicular to the x-axis.

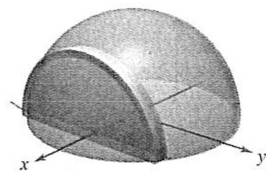
(a) Squares



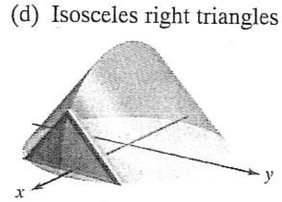
(b) Equilateral triangles



(c) Semicircles

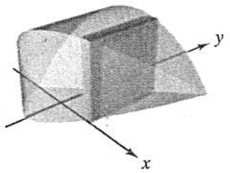


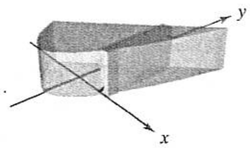
(d) Isosceles right triangles



56. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

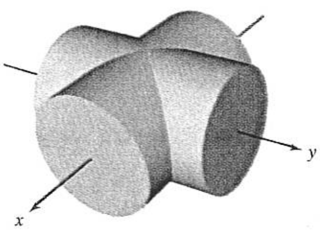
(a) Squares (b) Rectangles of height 1



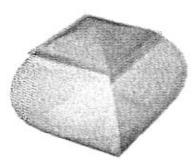


57. The base of a solid is bounded by $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid for each of the following cross sections (taken perpendicular to the y-axis): (a) squares, (b) semicircles, (c) equilateral triangles, and (d) semiellipses whose heights are twice the lengths of their bases.

58. Find the volume of the solid of intersection (the solid common to both) of the two right circular cylinders of radius r whose axes meet at right angles (see figure).



Two intersecting cylinders



Solid of intersection

FOR FURTHER INFORMATION For more information on this problem, see the article "Estimating the Volumes of Solid Figures with Curved Surfaces" by Donald Cohen in the May 1991 issue of *Mathematics Teacher*.

59. ***Volume of Oil*** A barrel of diameter d lies on a stand so that its axis makes an angle of 20° with its horizontal (see figure). The amount of motor oil in the barrel just covers the bottom of the barrel. Approximate the volume of oil.

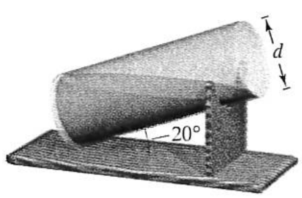
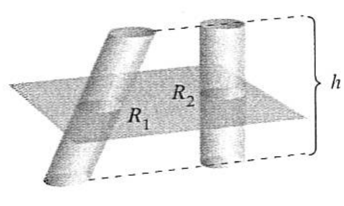


Figure for 59



Area of R_1 = area of R_2

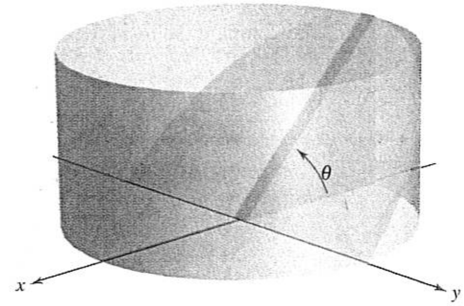
Figure for 60

60. ***Cavalieri's Theorem*** Prove that if two solids have equal altitudes and all plane sections parallel to their bases and at equal distances from their bases have equal areas, then the solids have the same volume (see figure).

61. Two planes cut a right circular cylinder to form a wedge. One plane is perpendicular to the axis of the cylinder and the second makes an angle of θ degrees with the first (see figure).

(a) Find the volume of the wedge if $\theta = 45°$.

(b) Find the volume of the wedge for an arbitrary angle θ. Assuming that the cylinder has sufficient length, how does the volume of the wedge change as θ increases from 0° to 90°?



62. A manufacturer drills a hole through the center of a metal sphere of radius R. The hole has a radius r. Find the volume of the resulting ring.

63. For the metal sphere in Exercises 62, let $R = 5$. What value of r will produce a ring whose volume is exactly half the volume of the sphere.

64. The solid shown in the figure has cross sections bounded by the graph of

$$|x|^a + |y|^a = 1$$

where $1 \le a \le 2$.

(a) Describe the cross section when $a = 1$ and $a = 2$.

(b) Describe a procedure for approximating the volume of the solid.