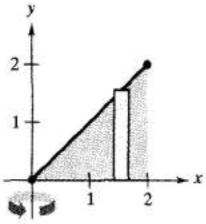
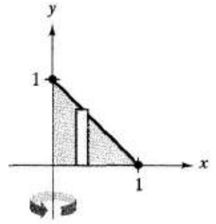
EXERCISES FOR SECTION 6.3

In Exercises 1–12, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

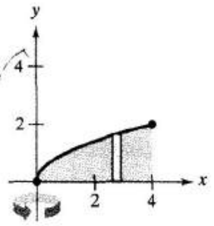
1. $y = x$



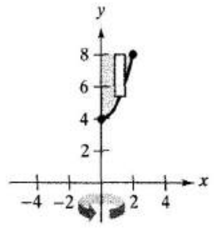
2. $y = 1 - x$



3. $y = \sqrt{x}$



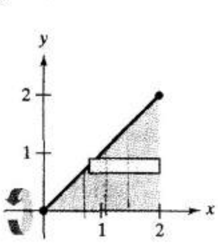
4. $y = x^2 + 4$



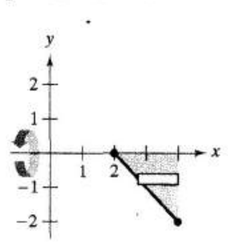
5. $y = x^2, y = 0, x = 2$
6. $y = x^2, y = 0, x = 4$
7. $y = x^2, y = 4x - x^2$
8. $y = 4 - x^2, y = 0$
9. $y = 4x - x^2, x = 0, y = 4$
10. $y = 2x, y = 4, x = 0$
11. $y = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}, y = 0, x = 0, x = 1$
12. $y = \begin{cases} \dfrac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}, y = 0, x = 0, x = \pi$

In Exercises 13–16, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

13. $y = x$



14. $y = 2 - x$



15. $y = \dfrac{1}{x}, x = 1, x = 2, y = 0$
16. $x + y^2 = 9, x = 0$

In Exercises 17–20, use the shell method to find the volume of the solid generated by revolving the plane region about the indicated line.

17. $y = x^2, y = 4x - x^2$, about the line $x = 4$
18. $y = x^2, y = 4x - x^2$, about the line $x = 2$
19. $y = 4x - x^2, y = 0$, about the line $x = 5$
20. $y = \sqrt{x}, y = 0, x = 4$, about the line $x = 6$

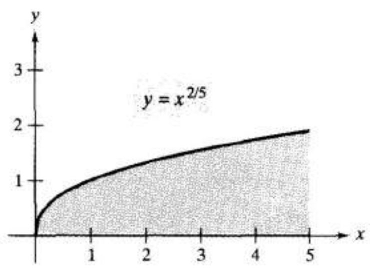
In Exercises 21–24, use the disc *or* the shell method to find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated line.

21. $y = x^3, y = 0, x = 2$
    (a) the x-axis (b) the y-axis (c) the line $x = 4$
22. $y = \dfrac{1}{x^2}, y = 0, x = 1, x = 4$
    (a) the x-axis (b) the y-axis (c) the line $y = 1$
23. $x^{1/2} + y^{1/2} = a^{1/2}, x = 0, y = 0$
    (a) the x-axis (b) the y-axis (c) the line $x = a$
24. $x^{2/3} + y^{2/3} = a^{2/3}, \quad a > 0$ (hypocycloid)
    (a) the x-axis (b) the y-axis

In Exercises 25–28, (a) use a graphing utility to graph the plane region bounded by the graphs of the equations and (b) use the integration capabilities of the graphing utility to approximate the volume of the solid generated by revolving the region about the y-axis.

25. $x^{4/3} + y^{4/3} = 1, x = 0, y = 0$, first quadrant
26. $y = \sqrt{1 - x^3}, y = 0, x = 0$
27. $y = \sqrt[3]{(x-2)^2(x-6)^2}, y = 0, x = 2, x = 6$
28. $y = \dfrac{2}{1 + e^{1/x}}, y = 0, x = 1, x = 3$

29. ***Think About It*** The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.
    (a) x-axis (b) y-axis (c) $x = 5$



30. Use integration to confirm your results in Exercise 29, where the region is bounded by the graphs of $y = x^{2/5}$, $y = 0$, and $x = 5$.

Think About It **In Exercises 31 and 32, determine which value best approximates the volume of the solid generated by revolving the region bounded by the graphs of the equations about the y-axis. (Make your selection on the basis of a sketch of the solid and *not* by performing any calculations.)**

31. $y = 2e^{-x}, y = 0, x = 0, x = 2$

(a) $\frac{3}{2}$ (b) -2 (c) 4 (d) 7.5 (e) 15

32. $y = \tan x, y = 0, x = 0, x = \dfrac{\pi}{4}$

(a) 3.5 (b) $-\frac{9}{4}$ (c) 8 (d) 10 (e) 1

33. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \frac{1}{2}x^2$ and $y = 2$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-fourth of the volume is removed. Find the diameter of the hole.

34. ***Machine Part*** A solid is generated by revolving the region bounded by $y = \sqrt{9 - x^2}$ and $y = 0$ about the y-axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole.

35. A hole is cut through the center of a sphere of radius r. The height of the remaining spherical ring is h, as shown in the figure. Show that the volume of the ring is $V = \pi h^3/6$. (*Note:* The volume is independent of r.)

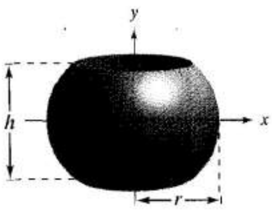


Figure for 35

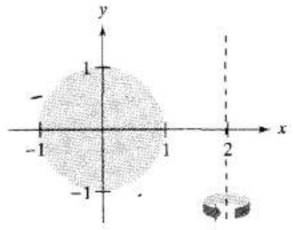


Figure for 36

36. ***Volume of a Torus*** A torus is formed by revolving the region bounded by the circle $x^2 + y^2 = 1$ about the line $x = 2$, as shown in the figure. Find the volume of this "doughnut-shaped" solid. (*Hint:* The integral $\int_{-1}^{1} \sqrt{1 - x^2}\,dx$ represents the area of a semicircle.)

37. ***Volume of a Torus*** Repeat Exercise 36 for a torus formed by revolving the region bounded by the circle $x^2 + y^2 = r^2$ about the line $x = R$, where $r < R$.

38. ***Volume of a Segment of a Sphere*** Let a sphere of radius r be cut by a plane, thus forming a segment of height h. Show that the volume of this segment is $\frac{1}{3}\pi h^2(3r - h)$.

Think About It **In Exercises 39 and 40, give a geometric argument that explains why the integrals have equal values.**

39. $\pi \int_1^5 (x - 1)\,dx$, $\quad 2\pi \int_0^2 y[5 - (y^2 + 1)]\,dy$

40. $\pi \int_0^2 [16 - (2y)^2]\,dy$, $\quad 2\pi \int_0^4 x\left(\dfrac{x}{2}\right) dx$

41. ***Think About It*** Match each of the integrals with the solid whose volume it represents, and give the dimensions of each solid.

(a) Right circular cone (b) Torus (c) Sphere
(d) Right circular cylinder (e) Ellipsoid

(i) $2\pi \int_0^r hx\,dx$

(ii) $2\pi \int_0^r hx\left(1 - \dfrac{x}{r}\right) dx$

(iii) $2\pi \int_0^r 2x\sqrt{r^2 - x^2}\,dx$

(iv) $2\pi \int_0^b 2ax\sqrt{1 - \dfrac{x^2}{b^2}}\,dx$

(v) $2\pi \int_{-r}^r (R - x)\left(2\sqrt{r^2 - x^2}\right) dx$

42. ***Volume of a Storage Shed*** A storage shed has a circular base of diameter 80 feet (see figure). Starting at the center, the interior height is measured every 10 feet and recorded in the table.

x	0	10	20	30	40
Height	50	45	40	20	0

(a) Use Simpson's Rule to approximate the volume of the building.

(b) Note that the roof line consists of two line segments. Find the equations of the line segments and use integration to find the volume of the shed.