EXERCISES FOR SECTION 7.7

Numerical and Graphical Analysis **In Exercises 1–4, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to support your result.**

1. $\lim_{x\to 0} \frac{\sin 5x}{\sin 2x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

2. $\lim_{x\to 0} \frac{1-e^x}{x}$

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$						

3. $\lim_{x\to\infty} x^5 e^{-x/100}$

x	1	10	10^2	10^3	10^4	10^5
$f(x)$						

4. $\lim_{x\to\infty} \frac{6x}{\sqrt{3x^2-2x}}$

x	1	10	10^2	10^3	10^4	10^5
$f(x)$						

In Exercises 5–10, evaluate the limit (a) using techniques from Chapters 1 and 3 and (b) using L'Hôpital's Rule.

5. $\lim_{x\to 3} \frac{2(x-3)}{x^2-9}$

6. $\lim_{x\to -1} \frac{2x^2-x-3}{x+1}$

7. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

8. $\lim_{x\to 0} \frac{\sin 4x}{2x}$

9. $\lim_{x\to\infty} \frac{5x^2-3x+1}{3x^2-5}$

10. $\lim_{x\to\infty} \frac{2x+1}{4x^2+x}$

In Exercises 11–30, evaluate the limit using L'Hôpital's Rule if necessary. (In Exercise 17, n is a positive integer.)

11. $\lim_{x\to 2} \frac{x^2-x-2}{x-2}$

12. $\lim_{x\to -1} \frac{x^2-x-2}{x+1}$

13. $\lim_{x\to 0} \frac{\sqrt{4-x^2}-2}{x}$

14. $\lim_{x\to 2^-} \frac{\sqrt{4-x^2}}{x-2}$

15. $\lim_{x\to 0} \frac{e^x-(1-x)}{x}$

16. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^3}$

17. $\lim_{x\to 0^+} \frac{e^x-(1+x)}{x^n}$

18. $\lim_{x\to 1} \frac{\ln x}{x^2-1}$

19. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

20. $\lim_{x\to 0} \frac{\sin ax}{\sin bx}$

21. $\lim_{x\to 0} \frac{\arcsin x}{x}$

22. $\lim_{x\to 1} \frac{\arctan x-(\pi/4)}{x-1}$

23. $\lim_{x\to\infty} \frac{3x^2-2x+1}{2x^2+3}$

24. $\lim_{x\to\infty} \frac{x-1}{x^2+2x+3}$

25. $\lim_{x\to\infty} \frac{x^2+2x+3}{x-1}$

26. $\lim_{x\to\infty} \frac{x^2}{e^x}$

27. $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$

28. $\lim_{x\to\infty} \frac{\sin x}{x-\pi}$

29. $\lim_{x\to\infty} \frac{\ln x}{x}$

30. $\lim_{x\to\infty} \frac{e^x}{x}$

In Exercises 31–44, (a) describe the type of indeterminate form (if any) that is obtained by direct substitution. (b) Evaluate the limit using L'Hôpital's Rule if necessary. (c) Use a graphing utility to graph the function and verify the result in part (b). (For a geometric approach to Exercise 31, see the article by John H. Mathews in the May 1992 issue of *The College Mathematics Journal*.)

31. $\lim_{x\to 0^+} (-x \ln x)$

32. $\lim_{x\to 0^+} x^2 \cot x$

33. $\lim_{x\to\infty} \left(x \sin \frac{1}{x}\right)$

34. $\lim_{x\to\infty} x \tan \frac{1}{x}$

35. $\lim_{x\to 0^+} x^{1/x}$

36. $\lim_{x\to 0^+} (e^x+x)^{1/x}$

37. $\lim_{x\to\infty} x^{1/x}$

38. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

39. $\lim_{x\to 0^+} (1+x)^{1/x}$

40. $\lim_{x\to\infty} (1+x)^{1/x}$

41. $\lim_{x\to 2^+} \left(\frac{8}{x^2-4}-\frac{x}{x-2}\right)$

42. $\lim_{x\to 2^+} \left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)$

43. $\lim_{x\to 1^+} \left(\frac{3}{\ln x}-\frac{2}{x-1}\right)$

44. $\lim_{x\to 0^+} \left(\frac{1}{x}-\frac{1}{x^2}\right)$

In Exercises 45–48, use a graphing utility to (a) graph the function and (b) find the required limit (if it exists).

45. $\lim_{x\to 3} \frac{x-3}{\ln(2x-5)}$

46. $\lim_{x\to 0^+} (\sin x)^x$

47. $\lim_{x\to\infty} \left(\sqrt{x^2+5x+2}-x\right)$

48. $\lim_{x\to\infty} \frac{x^3}{e^{2x}}$

49. ***Think About It*** Find the differentiable functions f and g that satisfy the specified condition such that $\lim_{x\to 5} f(x) = 0$ and $\lim_{x\to 5} g(x) = 0$.

(a) $\lim_{x\to 5} \frac{f(x)}{g(x)} = 10$ (b) $\lim_{x\to 5} \frac{f(x)}{g(x)} = 0$ (c) $\lim_{x\to 5} \frac{f(x)}{g(x)} = \infty$

(*Note:* There are many correct answers.)