Theorems Involving Differentiation

How do you prove that a functional value is the maximum or minimum?

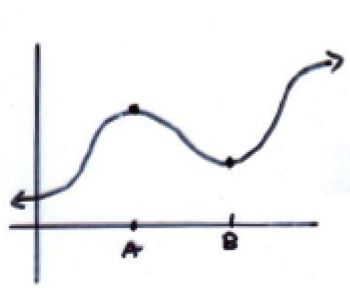
Extrema: $f(c)$ is the MINIMUM of f on [a,b] if $f(c) \leq f(x)$ for ALL x on [a,b] while $f(c)$ is the MAXIMUM of f on [a,b] if $f(c) \geq f(x)$ for ALL x on [a,b]

Extreme Value Theorem

If f is continuous on a closed interval [a,b], then f has both a minimum and a maximum on the interval.

Maximum and Minimum MUST Exist!

Relative Extrema: Maximum or Minimum that occur over an open interval

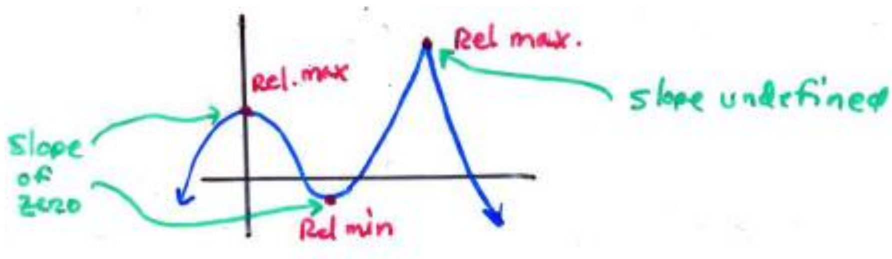


Relative maximum occurs at x = A and a
Relative minimum occurs at x = B

Note: When relative extrema are the extrema of the ENTIRE FUNCTION they would be called ABSOLUTE EXTREMA.

Where can relative extrema occur(a calculus reason)?

Relative extrema occur at a smooth turn(slope of tangent line is ZERO) or a sharp turn(slope of the tangent line is UNDEFINED).



Critical Number: an x-value of a function where the slope of the tangent line is either zero or undefined.

Using the Extreme Value Theorem for finding extrema on a closed interval [a,b]

Note that the function must be continuous on the interval [a,b]!

1) Evaluate $f(a)$ and $f(b)$

2) Find the **critical numbers** using the derivative. Evaluate those values in the function as well.

The largest of 1 and 2 is the MAXIMUM and the smallest is the MINIMUM.

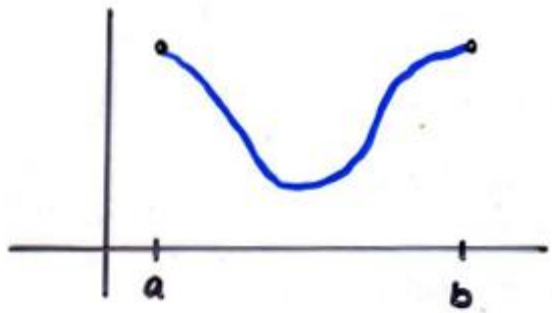
EX) Find all absolute extrema for each function over the given interval.

A) $f(x) = x^3 - 3x^2$ over [-2,1]

B) $f(x) = 2x - 3x^{2/3}$ over [-1,8]

Rolle's Theorem

Let f be continuous on a closed interval [a,b] AND differentiable on (a,b). If $f(a) = f(b)$ then there is at least one number c on (a,b) such that $f'(c)=0$.



Since ***differentiable***, the turn MUST happen at a horizontal tangent!

EX) Determine if Rolle's Theorem applies and if so, find all values for "c" such that $f'(c)=0$

A) $f(x)=x^3-3x^2$ over [0,3]

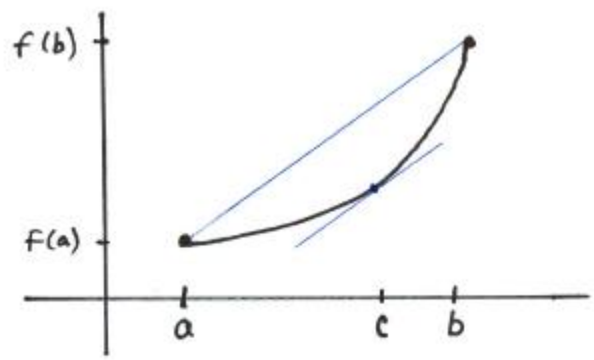
B) $f(x)=\dfrac{x}{x^2-2}$ over [-1,2]

The Mean Value Theorem (MVT)

If f is continuous on a closed interval [a,b] AND differentiable on (a,b) then there exists a number "c" on (a,b) such that:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

"Instantaneous ROC" = "Average ROC"



Note: Rolle's theorem is just a specific case of the MVT.

EX) Determine if the Mean Value Theorem applies and if so, find all values for "c"

such that $f'(c)=\frac{f(b)-f(a)}{b-a}$

A) $f(x)=x^3-x^2-2x$ over [-1,1]

B) $f(x)=x^{2/3}$ over [-1,8]