Find any critical numbers of the function.

1a) $f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x = 3x(x - 2)$

Critical numbers: $x = 0, 2$

1b) $h(x) = \sin^2 x + \cos x$

$0 < x < 2\pi$

1c) $f(x) = \frac{4x^2}{x+2}$

Show that the Extreme Value Theorem can be applied, them locate the absolute extrema of the function on the closed interval.

2a) $g(x) = x^2 - 2x,\ [0, 4]$

Continuous polynomial on a closed

interval so it can be applied!

$g'(x) = 2x - 2 = 2(x - 1)$

Critical number: $x = 1$

Left endpoint: $(0, 0)$

Critical number: $(1, -1)$ Minimum

Right endpoint: $(4, 8)$ Maximum

2b) $f(x) = x^3 - \frac{3}{2}x^2,\ [-1, 2]$

2c) $y = 3x^{2/3} - 2x, \ [-1, 1]$

2d) $f(x) = -|x-3| + 3, \ [-1, 5]$

2e) $f(x) = \begin{cases} 2x + 2, & 0 \le x \le 1 \\ 4x^2, & 1 < x \le 3 \end{cases}, \quad [0, 3]$

Determine whether Rolle's Theorem can be applied to on the closed interval [a, b]. If Rolle's Theorem can be applied, find all values of *c* in the open interval (a, b) such that $f'(c)=0$. If Rolle's Theorem cannot be applied, explain why not.

3a) $f(x) = -x^2 + 3x, \quad [0, 3]$

$f(0) = f(3) = 0$

f is continuous on $[0, 3]$ and differentiable on $(0, 3)$. Rolle's Theorem applies.

$f'(x) = -2x + 3$

$-2x + 3 = 0 \Rightarrow x = \frac{3}{2}$

c-value: $\frac{3}{2}$

3b) $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, \quad [-1, 3]$

3c) $f(x) = \sin x,\ [0, 2\pi]$ 3d) $f(x) = (x-1)^{2/3},\ [-2, 2]$ 3e) $f(x) = (x-3)(x+1)^2$, [-1, 3]

Determine whether the Mean Value Theorem can be applied to on the closed interval [a, b]. If the MVT can be applied, find all values of *c* in the open interval (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. If the MVT cannot be applied, explain why not.

4a) $f(x) = x^2,\quad [-2, 1]$

$f(x) = x^2$ is continuous on $[-2, 1]$ and differentiable on $(-2, 1)$.

$$\frac{f(1) - f(-2)}{1 - (-2)} = \frac{1 - 4}{3} = -1$$

$$f'(x) = 2x = -1$$

$$x = -\frac{1}{2}$$

$$c = -\frac{1}{2}$$

4b) $f(x) = x^{2/3},\quad [0, 1]$

4c) $f(x)=|x+1|-2$, [-3, 3]

4d) $f(x) = \sqrt{2-x}, \quad [-7, 2]$

5) AP MULTIPLE CHOICE EXAMPLES

1) If f is continuous for $a \le x \le b$ and differentiable for $a < x < b$, which of the following could be false?

(A) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ for some c such that $a < c < b$.

(B) $f'(c) = 0$ for some c such that $a < c < b$.

(C) f has a minimum value on $a \le x \le b$.

(D) f has a maximum value on $a \le x \le b$.

2) Let f be the function given by $f(x) = x^3 - 3x^2$. What are all values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0, 3]$?

(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

3) GRAPHING CALCULATOR ALLOWED

The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0,10)$?

(A) One
(B) Three
(C) Four
(D) Five
(E) Seven

4) The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$
(B) $(1,1)$
(C) $(2,\sqrt{2})$
(D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$
(E) None of the above