AP Calc Using Properties & Geometry to Evaluate Definite Integrals Answers

1b) Given $\int_2^6 f(x)\,dx = 10$ and $\int_2^6 g(x)\,dx = -2$, evaluate

(a) $\int_2^6 [f(x) + g(x)]\,dx.$ (b) $\int_2^6 [g(x) - f(x)]\,dx.$

(c) $\int_2^6 2g(x)\,dx.$ (d) $\int_2^6 3f(x)\,dx.$

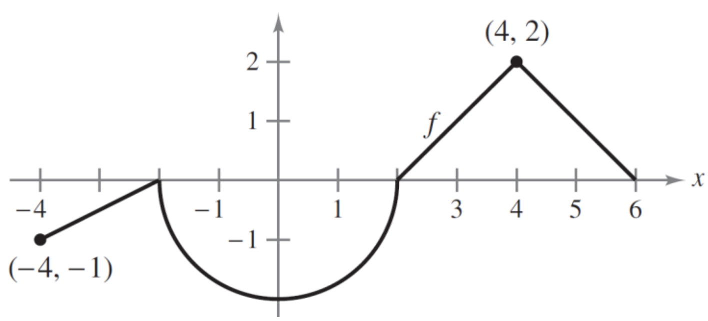
(a) $\int_2^6 [f(x) + g(x)]\,dx = \int_2^6 f(x)\,dx + \int_2^6 g(x)\,dx$
$= 10 + (-2) = 8$

(b) $\int_2^6 [g(x) - f(x)]\,dx = \int_2^6 g(x)\,dx - \int_2^6 f(x)\,dx$
$= -2 - 10 = -12$

(c) $\int_2^6 2g(x)\,dx = 2\int_2^6 g(x)\,dx = 2(-2) = -4$

(d) $\int_2^6 3f(x)\,dx = 3\int_2^6 f(x)\,dx = 3(10) = 30$

1c)



(a) $\int_0^2 f(x)\,dx$ (b) $\int_2^6 f(x)\,dx$ (c) $\int_{-4}^2 f(x)\,dx$

(d) $\int_{-4}^6 f(x)\,dx$ (e) $\int_{-4}^6 |f(x)|\,dx$ (f) $\int_{-4}^6 [f(x) + 2]\,dx$

(a) Quarter circle below x-axis:
$-\frac{1}{4}\pi r^2 = -\frac{1}{4}\pi(2)^2 = -\pi$

(b) Triangle: $\frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$

(c) Triangle + Semicircle below x-axis:
$-\frac{1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 = -(1 + 2\pi)$

(d) Sum of parts (b) and (c): $4 - (1 + 2\pi) = 3 - 2\pi$

(e) Sum of absolute values of (b) and (c):
$4 + (1 + 2\pi) = 5 + 2\pi$

(f) Answers to (d) plus
$2(10) = 20$: $(3 - 2\pi) + 20 = 23 - 2\pi$

1d) Given $\int_0^5 f(x)\,dx = 4.$

Evaluate each integral.

(a) $\int_0^5 [f(x) + 2]\,dx$ (b) $\int_{-2}^3 f(x+2)\,dx$

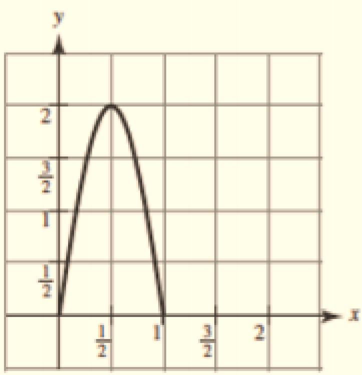
(a) $\int_0^5 [f(x) + 2]\,dx = \int_0^5 f(x)\,dx + \int_0^5 2\,dx$

$= 4 + 10 = 14$

(b) $\int_{-2}^3 f(x+2)\,dx = \int_0^5 f(x)\,dx = 4$ (Let $u = x + 2$.)

2b) $\int_0^1 2 \sin \pi x\,dx$

(a) 6 (b) $\frac{1}{2}$ (c) 4 (d) $\frac{5}{4}$



$\int_0^1 2 \sin \pi x\,dx \approx \frac{1}{2}(1)(2) \approx 1$

So, answer is “D”(5/4) since “1” represents the area of the triangle used to underestimate the exact integral.

3) AP MULTIPLE CHOICE EXAMPLES

1) If $\int_1^{10} f(x)\,dx = 4$ and $\int_{10}^{3} f(x)\,dx = 7$, then $\int_1^{3} f(x)\,dx =$

(A) -3 (B) 0 (C) 3 (D) 10 $\boxed{\text{(E) } 11}$

$$\int_{10}^{3} f(x)\,dx = -\int_3^{10} f(x)\,dx$$

NOTE: $7 = -\int_3^{10} f(x)\,dx$

(by property) $-7 = \int_3^{10} f(x)\,dx$

$$\int_1^3 f(x)\,dx = \int_1^{10} f(x)\,dx - \int_3^{10} f(x)\,dx$$

$$\int_1^3 f(x)\,dx = 4 - (-7)$$

$$= \boxed{11}$$

2) If $\int_a^b f(x)\,dx = 2a - 5b$, then $\int_a^b [f(x) - 2]\,dx =$

(A) $-7b$ (B) $-3b$ (C) $4a - 7b$ (D) $4a - 3b$

$$\int_a^b [f(x) - 2]\,dx$$

$$= \int_a^b f(x)\,dx - \int_a^b 2\,dx$$

$$= 2a - 5b - [2 \cdot (b - a)]$$

$$= 2a - 5b - [2b - 2a]$$

$$= 2a - 5b - 2b + 2a$$

$$= \boxed{4a - 7b}$$